

**COMPARISON OF VOID FRACTION  
CORRELATIONS FOR TWO-PHASE  
FLOW IN HORIZONTAL AND  
UPWARD INCLINED FLOWS**

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1998**

**Submitted to the Faculty of the  
Graduate College of  
Oklahoma State University  
in partial fulfillment of  
the requirements for  
the degree of  
MASTER OF SCIENCE  
May 2006**

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## ACKNOWLEDGEMENTS

I would like to express my gratitude to my advisor Dr. A.J. Ghajar for his untiring support and guidance not only in this work and preparation of the manuscript but also throughout my entire graduate study.

My sincere thanks go to Dr. F.W. Chambers and P.M. Moretti for being in my committee and also their kind advice during my masters program.

No words would be good enough to express my gratitude to my parents and all my family who have done everything to make my graduate studies possible. I will forever cherish their endless love and support.

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## NOMENCLATURE

$A$	Constant in Butterworth (1975) correlation
$A_{PRM}$	Premoli et al. (1970) correlation constant
$A_{SM}$	Constant in Smith (1969) correlation
$b$	Constant in Butterworth (1975) correlation
$B_{gas}$	Gas formation volume factor, ft <sup>3</sup> /scf
$c$	Constant in Butterworth (1975) correlation
$C$	Inclination factor constant in Beggs (1972) correlation
$C_{ABD}$	Abdulmajeed (1996) correction factor
$C_0$	Distribution parameter
$C_{MC1}$	Constant in Maier and Coddington (1997) correlation
$C_{MC2}$	Constant in Maier and Coddington (1997) correlation
$d$	Constant in Butterworth (1975) correlation
$d_1$	Factor in Moussali correlation as given by Isbin and Biddle (1979)
$D$	Diameter of pipe
$D_H$	Hydraulic diameter of vessel
$D_H^*$	Dimensionless hydraulic diameter, $\frac{D_H}{\left(\frac{\sigma}{\rho_L - \rho_G}\right)^{0.5}}$
$D_v$	Wilson et al. (1961) correlation dimensionless number, $\frac{\rho_G}{\rho_L - \rho_G}$
$E_L$	Liquid-holdup

$F$	Wilson et al. (1961) Froude number, $U_{SG} \left[ \frac{\rho_L - \rho_G}{g\sigma} \right]^{0.25}$
$F_D$	Constant in Gardner (1980) correlation, $F_D = \frac{U_{SG} \rho_L^{1/2}}{[(\rho_L - \rho_G)g\sigma]^{1/4}}$
$F_1$	Constant in Premoli et al. (1970) correlation
$F_2$	Constant in Premoli et al. (1970) correlation
$Fr$	Froude number, $\frac{U_M^2}{gD}$
$Ft$	Froude rate parameter
$F(X_{tt})$	Constant in Tandon et al. (1985) correlation, $0.15 \left( \frac{1}{X_{tt}} + \frac{2.85}{X_{tt}^{0.476}} \right)$
$g$	Acceleration of gravity
$G$	Mass flux
$j_g^*$	Dimentiev et al. correlation gas flux as reported by Kataoka and Ishii  (1987), $\frac{\rho_G^{0.5} U_{SG}}{[gD(\rho_L - \rho_G)]^{0.5}}$
$j_L^*$	Kawaji et al. (1987) correlation dimensionless liquid flux,  $\frac{\rho_L^{0.5} U_{SL}}{[gD(\rho_L - \rho_G)]^{0.5}}$
$K$	General coefficient
$K_B$	Bankoff (1960) parameter
$K_{Fj}$	Constant in Fujie (1964) correlation
$K_{GR}$	Constant in Gardner (1980) correlation (Either 1.7 or 11.2)
$K_{HO}$	Hoogendoorn (1959) correlation constant
$K_{HU}$	Hughmark (1962) parameter

$K_{MO}$	Moussali constant as given by Isbin and Biddle (1979)
$l$	Constant in Abdulmajeed (1996) correlation
$L$	Laplace length scale defined in Gardner (1980) correlation, $\left[ \frac{\sigma}{(\rho_L - \rho_G)g} \right]$
$m$	Mass
$N_{FR}$	Froude number in Beggs (1972) correlation, $Fr$
$N_{GV}$	Gas velocity number, $U_{SG} \left[ \frac{\rho_L}{g\sigma} \right]^{0.25}$
$N_L$	Liquid viscosity number, $\frac{\mu_L}{[\rho_L \sigma^3]^{0.25}}$
$N_{LV}$	Liquid velocity number, $U_{SL} \left[ \frac{\rho_L}{g\sigma} \right]^{0.25}$
$P$	System pressure
$P_C$	Critical pressure (thermodynamic)
$R$	Parameter in Abdulmajeed (1996) correlation
$Re$	Reynolds number
$Re_L$	Reynolds number, liquid, $\frac{GD(1-x)}{\mu_L}$
$R_L$	Liquid holdup
$Re_M$	Mixture Reynolds number, $\frac{\rho_L U_M D}{\mu_L}$
$Re_{SL}$	Superficial liquid Reynolds number, $\frac{\rho_L U_{SL} D}{\mu_L}$
$S$	Slip ratio
$U_G$	Velocity of gas

$U_{GM}$	Drift velocity, $U_G - U_M$
$U_L$	Velocity of liquid
$U_M$	Mixture velocity, $U_{SL} + U_{SG}$
$U_{SG}$	Superficial gas velocity
$U_{SL}$	Superficial liquid velocity
$v_1$	Constant in Maier and Coddington (1997) correlation
$v_2$	Constant in Maier and Coddington (1997) correlation
$v_3$	Constant in Maier and Coddington (1997) correlation
$v_4$	Constant in Maier and Coddington (1997) correlation
$v_5$	Constant in Maier and Coddington (1997) correlation
$v_6$	Constant in Maier and Coddington (1997) correlation
$We_L$	Weber number, liquid
$x$	quality, mass of vapor/total mass
$X_{tt}$	Lockhart - Martinelli (1949) Parameter, $X_{tt} = \left( \frac{\mu_L}{\mu_G} \right)^{0.1} \left( \frac{1-x}{x} \right)^{0.9} \left( \frac{\rho_G}{\rho_L} \right)^{0.5}$
$y$	Constant in Premoli et al. (1970) correlation
$Z$	Parameter in Hughmark (1962) correlation
$Z_1$	Parameter in Minami and Brill (1987) correlation
$K$	Parameter in Lee et al. correlation (Jeje and Mattar 2004)
$K_1$	Constant in Lee et al. correlation (Jeje and Mattar 2004)
$K_2$	Constant in Lee et al. correlation (Jeje and Mattar 2004)
$K_3$	Constant in Lee et al. correlation (Jeje and Mattar 2004)
$K_4$	Constant in Lee et al. correlation (Jeje and Mattar 2004)

$K_5$	Constant in Lee et al. correlation (Jeje and Mattar 2004)
$T$	Temperature
$P$	Pressure
$R$	The universal gas constant (8314.34 J/ kmol.K)
$R^2$	Correlation coefficient
$M$	Molecular weight
$U_{SG}$	Superficial gas velocity
$U_M$	Mixture velocity, $U_{SL} + U_{SG}$
$X$	Parameter in Lee et al. correlation (Jeje and Mattar 2004)
$x_1$	Constant in Lee et al. correlation (Jeje and Mattar 2004)
$x_2$	Constant in Lee et al. correlation (Jeje and Mattar 2004)
$x_3$	Constant in Lee et al. correlation (Jeje and Mattar 2004)
$Y$	Parameter in Lee et al. correlation (Jeje and Mattar 2004)
$y_1$	Constant in Lee et al. correlation (Jeje and Mattar 2004)
$y_2$	Constant in Lee et al. correlation (Jeje and Mattar 2004)
$Z$	Compressibility factor

#### Greek Letters

$\alpha$	Void fraction, average
$\alpha_H$	No slip (homogeneous) void fraction
$\beta$	Volumetric quality, $\alpha_H$
$\lambda$	Input liquid content, $\frac{U_{SL}}{U_{SL} + U_{SG}}$
$\mu$	Viscosity

$\rho$	Density
$c$	Surface tension
$\theta$	Pipe inclination angle

#### Subscripts

$kero$	Kerosene
$pc$	Pseudo-critical (pressure, temperature)
$r$	Reduced (pressure, temperature)
$G$	Gas phase
$L$	Liquid phase
$M$	Mixture
$N_{gas}$	Natural gas
$S$	Superficial



## **CHAPTER 1**

### **INTRODUCTION**

Two phase flow, theoretically, is the simultaneous flow of two of any of the three discrete phases (solid, liquid or gas) of any substance or combination of substances. Practical applications of a gas-liquid flow, of a single substance or two different components, are commonly encountered in the petroleum, nuclear and process industries. The two phases may be of different components and/or there could be a phase change due to evaporation and condensation of a single fluid.

Considerable effort has been devoted to understanding the underlying physics of two phase flow in the past. Theoretical and empirical correlations have been developed for predictions of the various parameters of practical interest such as flow pattern transitions, pressure drop and void fraction.

In the industrial applications where two phase flow exists, the task of sizing the equipment for gathering, pumping, transporting and storing such a two phase mixture requires the formidable task of predicting the phase distribution in the system from given operating conditions. Once this phase distribution is known, the problem may be simplified in such a way that it could be approached and tackled in a similar fashion analogous to single phase flow.

One of the critical unknown parameters involved in predicting the pressure loss and heat transfer (see for example Kim and Ghajar,2006) in any gas-liquid system is the void fraction or liquid holdup which is the volume of the space occupied by the gas or

liquid respectively. The seemingly benign issue of determining the phase distribution from input conditions in a given pipe is complicated as a result of the slippage between the gas and the liquid phases.

Void fraction data was measured by different researchers in the effort for development of the numerous empirical correlations and also to validate some of the theoretical work pursued in analyzing two phase flow behavior.

To accomplish the task of measuring void fraction, an appreciable number of experimental facilities have been built and tests run in order to measure void fraction under different operating conditions. The experimental set ups have been designed and constructed ranging in size from very short, small diameters to those which simulate big petroleum field conditions. As expected, these setups are operated with different fluids under a broad range of temperatures, pressures and pipe inclinations spanning from vertically upwards to vertically downwards.

A huge number of studies have been carried out and are still on going in an effort to accurately predict the void fraction from the known operating conditions. These analyses range in scope from a fully theoretical one for the simple cases to the fully empirical methods when the interaction between the phases becomes complex. Owing to the complexity and lack of understanding of the basic underlining physics of the problem, the majority of the analyses are more inclined towards empirical correlations. For the sake of simplicity both the theoretical models as well as the empirical correlations that are collected from the literature and discussed here would be simply referred to as void fraction correlations.

The design engineer is faced with the difficult task of choosing the “right” correlation among the plethora of correlations available. The fact that there are plenty of correlations available would not be a concern had it not been for the fact that most of

the correlations have some form of restrictions attached to them. The lack of understanding of the exact behavior of two phase flow had led many investigators to resort to developing a correlation which only works for a given range of flow rates, holdup values or flow patterns.

The idea of combining different correlations which claim to do well for the specific flow characteristics for which they are developed in trying to take care of all the design need would entail its own difficulty. For instance, one of the most common restrictions to the correlations, flow pattern dependency, is sometimes a purely subjective judgment of the investigator especially for those points on or near flow pattern boundaries. Moreover, one is bound to run into some discontinuity when switching from one correlation to the other for different practical operating conditions.

The other pitfall is that the majority of the void fraction data and also the correlations are primarily developed for mostly horizontal orientation though some vertical flows and inclined cases have been investigated.

The purpose of this work is to find a void fraction correlation that could acceptably predict most of the experimental data collected for all inclination angles as well as the different flow patterns without resorting back to too much complex expressions that require iterative schemes with multiple solutions.

A search is made in the open literature to collect all the available void fraction correlations as well as the measured void fraction data from various sources.

Comparisons of predictive performance and promise of the capability of the collected void fraction correlations was made using the collected data from the literature.

Based on the analysis, the best performing correlations were selected and recommendations were drawn on their strengths and weaknesses. A detailed study of

the performance of several of the top performing correlations together with the data points which they could not handle well was made. Introducing correction factors which take into account the physical nature of the data sets, an improved version to one of these correlations was suggested that could handle data points which failed to be captured by the original correlation.

This work has been organized in such a way that, first, the void fraction correlations which are collected from the literature would be presented briefly in Chapter 2. Moreover, most of the void fraction comparison work carried out by different researchers at different times is also presented.

Chapter 3 will be concerned with the presentation of the experimental data base collected from the literature along with the associated fluid physical properties of the working fluids used in the tests.

The detail predictive capability of each of the void fraction correlations reported in Chapter 2 in capturing the experimental data base compiled (Chapter 3) will be investigated and results of the comparisons between the correlations would be discussed in Chapter 4. Steps on how we have developed an improved version of one of the best performing correlations would be given as an extension in Chapter 4.

Finally, concise conclusions and recommendation based on the overall undertaking are forwarded in Chapter 5.

## CHAPTER 2

### LITERATURE REVIEW

The extensive literature review presented here had two major parts that were undertaken in parallel. The primary task was to find most if not all of the void fraction correlations that are available in the open literature while the other one was gathering a sufficiently large number of experimental void fraction data that covered a wide range of fluids and operating conditions such as pipe size, flow rates and the associated flow patterns, temperature, pressure and different inclination angles.

This chapter would focus on the discussion of the collected void fraction correlations and some of the previous comparison work done. The experimental data base and characteristics related to it would be discussed in the next chapter.

In facilitating better understanding of this manuscript, it is worth while to highlight some of the most common terminologies and definitions of parameters that would be encountered throughout this work.

Void fraction is defined as the volume of space the gas phase occupies in a given two phase flow in a pipe, hence for a total pipe cross sectional area of  $A$ ; the void fraction is given by

$$\alpha = \frac{A_G}{A}$$

Liquid holdup is the complement of the void fraction in the pipe, i.e., it is the remaining volume of space occupied by the liquid phase. Thus, liquid holdup is

$$R_L = 1 - \alpha = \frac{A_L}{A}$$

The quality of the mixture,  $x$ , in the isothermal flow case we are considering here is taken as the input mass of the gaseous phase to that of the total mixture mass of  $m$ , hence

$$x = \frac{m_G}{m_M}$$

The slip ratio,  $S$ , is defined as the ratio of the actual velocities between the phases. A slip ratio of unity for a mixture being the homogeneous case where it is assumed that both phases travel at the same velocity. The slip ratio is defined as

$$S = \frac{U_G}{U_L}$$

The superficial gas,  $U_{SG}$ , and liquid,  $U_{SL}$ , velocities are defined as the velocities of the gas or liquid phase in the pipe assuming the flow is a single phase in either gas or liquid respectively.

From the definitions given above and writing conservation of mass for each phase and total flow, we can define the relationships,

$$U_G = \frac{U_{SG}}{\alpha}$$

$$U_L = \frac{U_{SL}}{1 - \alpha} \quad \text{and}$$

$$\frac{x}{1 - x} = \left( \frac{\rho_G}{\rho_L} \right) \left( \frac{U_{SG}}{U_{SL}} \right)$$

Having given the basic definitions of the most important and frequently used parameters in two phase flow in relation to void fraction, we now move on to presenting the correlations.

## **2.1 Void Fraction Correlations**

Different void fraction (liquid holdup) correlations which appeared from the early 1940s to date collected from the open literature are presented here. The total number of correlations collected is more than 80. We have presented correlations for which we have complete information and those with little or no information at all but which we found to perform well in predicting a good portion of the data points in our database. These correlations were developed from theoretical and mostly experimental investigations under various operating conditions.

In presenting the correlations, different criteria were sought into which the developed correlations might conveniently fall. One advantage of it is that it would shed some light on the common features or lack thereof between individual and groups of correlations. There are a number of simple criteria that could easily be used to categorize these correlations. However, their shortfalls would heavily outweigh the practical advantage one would get out of them. We have briefly discussed some of these criteria and their associated shortfalls below.

The majority of the correlations were developed from horizontal test section experiments with only a few for other inclination angles, the common one being upward  $90^0$ . Categorizing the correlations along their applicability with regard to angle of inclination would not serve any purpose as most of them would fall under the horizontal case.

As the complex nature of two phase flow has not yielded to any theoretical formulation of a particular flow pattern analysis let alone a general one, it can be observed from a literature search that the trend in correlation development is becoming more geared towards a specific area of particular interest which in effect has resulted in correlations for a specific flow regime. Hence, division along the type

of flow pattern dependency is a logical step. Due to the highly subjective and tricky nature of assigning a specific flow pattern to a given flow phenomenon, we have avoided lumping the correlations to a specific flow pattern even when the applicability of the correlations has been stated so by the investigator. It is assumed that the correlation is a general type if no specific restriction is implied by the original author or no discussion thereof is provided in the reference in which it was reported. However, a flow pattern specific correlation would therefore be reported so in order to avoid unfair and misleading judgment during the predictive performance assessment. Apart from the two criteria discussed above, types of fluid considered, pipe diameters and mass flow rates are the other areas where the correlations would share some form of commonality. Most of the data from which the correlations have been developed was from small diameter, short length pipes in a laboratory setting with controlled, and relatively small mass flow rates while mixtures of air-water dominate with regard to the fluids considered. Even though these factors have significant influence in the predictive capability of a correlation, it was decided not to group them along these lines as the majority of the correlations would aggregate into a single category. Finally, it is imperative to mention here that by nature all the empirical correlations have some sort of limitations, even though not explicitly reported by their authors. This is due to the very fact that they were fitted to given data sets which in turn depend on the inherent physical limitations of the experiment under which the data was collected. Therefore, rather than going after physical parameters (inclination angles, mass flow rates etc) which are too narrow or subjective criteria like flow pattern upon which there is no firm consensus to this date, we have decided to follow the work of Vijayan et al.(2000) to classify the correlations into four categories. These are:



1. Slip ratio correlations
2.  $Ka_H$  correlations
3. Drift flux correlations
4. General void fraction correlations

We shall now give a brief description of these categories and the correlations that fall into each of them.

### 2.1.1 Slip Ratio Correlations

These correlations are of the form

$$\alpha = \frac{1}{1 + S \left( \frac{1-x}{x} \right)^b \left( \frac{\rho_G}{\rho_L} \right)^c}$$

Where the slip ratio S and the general expression has been shown by Butterworth (1975) to take the form

$$\alpha = \frac{1}{1 + A \left( \frac{1-x}{x} \right)^b \left( \frac{\rho_G}{\rho_L} \right)^c \left( \frac{\mu_L}{\mu_G} \right)^d}$$

Discussion of some of the notable void fraction correlations that fall in this category follows. The constants (A, b, c, d) in the above equation for the different correlations are given in Table 1.

The most simple of all the correlations with a theoretical background and the assumption that the gas and liquid velocities are equal or there is no slip between them is the **Homogeneous** model or the no-slip correlation.

It is almost impossible to find any literature on void fraction correlations which would not refer to the pioneer work and the void fraction correlation of **Lockhart** and **Martinelli** (1949). The correlation was in a plot form where the void fraction was correlated with what is now commonly known as the Lockhart-Martinelli Parameter,  $X_{tt}$ . The empirical approximation of this graphical representation given by Butterworth (1975) is used in this analysis.

The correlation by **Fauske** (1961) can also be shown to fall into a similar expression form. A complex expression between steam quality and void fraction was given by **Fujie** (1964) for horizontal and vertical flows with and without heating. The isothermal correlation for the horizontal case is considered here which can be reduced and cast into the slip ratio model form. It must be noted here that the constant term A in the Butterworth (1975) expression is a function of the void fraction and hence the whole process requires an iterative solution.

The work of Butterworth (1975) showed the striking similarity in variables between six of the correlations he considered. He made approximate transformations of the graphical representations of the original authors and expressed it with an appropriate equation. The correlations reported in Butterworth (1975) in addition to the Homogeneous and Lockhart-Martinelli correlations are that of **Thom** (1964) for which the “slip factor” plot was approximated by a parameter which is a function of the density and viscosity ratios of the two phases and **Zivi** (1964) which was developed from the principle of minimum entropy production for steam void fraction. The correlation by **Moody** (1965) which in the limit for the maximum flow rate, is exactly identical to that of **Zivi** (1964) has been taken out of the analysis. The **Turner** and **Wallis** (1965) separate cylinders model for turbulent-turbulent flow has also been cast in the slip ratio form. The **Baroczy** (1966) correlation which is in graphical form

has also been approximated to fit into the general type of slip ratio correlation presented to show the similarity between the correlations.

**Smith** (1969) developed a correlation based on equal velocity heads of the homogeneous mixture core and the annulus liquid phase. The expression for the correlation is

$$\alpha = \frac{1}{\left[ 1 + \left( \frac{\rho_G}{\rho_L} \right) \left( \frac{1-x}{x} \right)^* \left\{ 0.4 + 0.6 \sqrt{\frac{\left( \rho_L / \rho_G \right) + 0.4 \left( \frac{1-x}{x} \right)}{1 + 0.4 \left( \frac{1-x}{x} \right)}} \right\} \right]}$$

where the constant in Table 1 is given as  $A_{SM} = \left( 0.4 + 0.6 \sqrt{\frac{\left( \frac{\rho_L}{\rho_G} \right) + 0.4 \left( \frac{1-x}{x} \right)}{1 + 0.4 \left( \frac{1-x}{x} \right)}} \right)$

**Premoli et al.** (1970) correlation or sometimes referred to as the **CISE** correlation is developed from a similar analysis as the homogeneous model introducing the Weber and Reynolds numbers for different conditions of two phase flow in vertical adiabatic channels. The correlation is given by

$$\left( \frac{1-\alpha}{\alpha} \right) \left( \frac{x}{1-x} \right) \left( \frac{\rho_L}{\rho_G} \right) = 1 + F_1 \left\{ \frac{y}{1+yF_2} - yF_2 \right\}^{1/2}$$

Where, the different coefficients in the expression can be calculated via

$$F_1 = 1.578 \text{Re}_L^{-0.19} \left( \frac{\rho_L}{\rho_G} \right)^{0.22}$$

$$F_2 = 0.0273 \text{We}_L \text{Re}_L^{-0.51} \left( \frac{\rho_L}{\rho_G} \right)^{-0.08}$$

$$y = \frac{\beta}{1-\beta} \quad \text{Re}_L = \frac{GD}{\mu_L} \quad \text{We}_L = \frac{G^2 D}{\sigma \rho_L g} \quad \beta = \frac{1}{1 + \left( \frac{1-x}{x} \right) \left( \frac{\rho_G}{\rho_L} \right)}$$

where the constant in Table 1 is given as  $A_{PRM} = 1 + F_1 \left\{ \frac{y}{1 + yF_2} - yF_2 \right\}^{1/2}$

The **Chisholm** (1973) correlation also falls into this category. **Madsen** (1975) developed a void fraction correlation for the bulk boiling phenomenon where many flow regimes were encountered. **Spedding** and **Chen** (1984) gave a correlation for void fraction values of 0.8 and greater for horizontal air-water flow data. A further extension of the Spedding and Chen correlation by **Chen** (1986) is also included.

The correlation by **Hamersma** and **Hart** (1987) which can be cast in the same expression and with coefficients given for their experimental air-water data is considered. This correlation has been developed for liquid holdups less than 0.04. The **Petalaz** and **Aziz** (1997) correlation as reported in Ouyang and Aziz (2002) is primarily for annular mist flow and also can be cast in the same form. **Zhao et al.** (2000) correlation was developed using 189 (refined from 255) sets of geothermal experimental data points (steam and water) and using the seventh power law velocity distribution.

### 2.1.2 $Ka_H$ Correlations

The other category of void fractions are those which are a constant multiple or some function (Isbin and Biddle, 1979) of the no slip(homogeneous) void fraction correlation.

In an effort to predict the void fraction taking into consideration the non homogeneous nature of the two phase flow, **Armand** (1946) gave a correlation in the early days of two phase flow research and is one of the first few correlations to be developed and is given as

$$\alpha = 0.833\alpha_H$$

The modifications by **Chisholm** (1983) -**Armand**(1946) is given by

$$\alpha = \frac{1}{\alpha_H + (1 - \alpha_H)^{0.5}} \alpha_H$$

**Bankoff** (1960) correlation is developed from analysis on a single fluid with variable density and velocity profile for vertical flow. The original form of the equation is

$$\frac{1}{x} = 1 - \frac{\rho_L}{\rho_G} \left( 1 - \frac{K_B}{\alpha} \right)$$

where,  $K_B = 0.71 + (1.45 \times 10^{-2})P$ ,  $P$  in MPa

Upon rearrangement of which becomes,

$$\alpha = K_B \alpha_H$$

**Hughmark** (1962) correlation is a modification of the Bankoff (1960) correlation primarily targeted at improving the applicability of the correlation to two phase flows other than steam water. With a new flow parameter  $Z$  defined in terms of the Reynolds and Froude numbers and a no slip liquid volume fraction, it was claimed that the correlation works for both vertical as well as horizontal flows. Hughmark modified the flow parameter to be characterized as,

$$K_{HU} = f(Z)$$

where

$$Z = \frac{Re^{1/6} Fr^{1/8}}{\lambda^{1/4}} \quad Re = \frac{GD}{(1 - \alpha)\mu_L + \alpha\mu_G} \quad Fr = \frac{U_M^2}{gD} \quad \lambda = \frac{1}{1 + \frac{x}{1 - x} \frac{\rho_L}{\rho_G}}$$

The correlation by **Nishino** and **Yamazaki**(1963) is given as

$$\alpha = 1 - \left[ \frac{1 - x}{x} \frac{\rho_G}{\rho_L} \right]^{0.5} (\alpha_H)^{0.5}$$

The correlation presented in 1964 by **Kowalczewski** that was reported by Isbin and Biddle(1979) is given as

Table 1 Slip Ratio Correlations Coefficients for Butterworth's (1975) Correlation

Correlation	A	b	c	d
Homogeneous	1	1	1	0
Lockhart & Martinelli(1949)	0.28	0.64	0.36	0.07
Fauske(1961)	1	1	0.5	0
Fujie(1964)	$\sqrt{K_{FJ}\alpha + 1}$	1	1	0
Thom(1964)	1	1	0.89	0.18
Zivi (1964)	1	1	0.67	0
Turner & Wallis(1965)	1	0.72	0.4	0.08
Baroczy(1966)	1	0.74	0.65	0.13
Smith(1969)	$A_{SM}$	1	1	0
Premoli et al.(1970)	$A_{PRM}$	1	1	0
Chisholm(1973)	$\sqrt{1-x\left(1-\frac{\rho_L}{\rho_G}\right)}$	1	1	0
Madsen(1975)	1	$1 + \frac{\log\left(\frac{\rho_L}{\rho_G}\right)}{\log\left(\frac{1-x}{x}\right)}$	-0.5	0
Spedding & Chen(1984)	2.22	0.65	0.65	0
Chen(1986)	0.18	0.6	0.33	0.07
Hamersma & Hart(1987)	0.26	0.67	0.33	0
Petalaz & Aziz(1997)	$0.735\left(\frac{\mu_L^2 U_{SG}^2}{\sigma^2}\right)^{0.074}$	-0.2	-0.126	0
Zhao et al.(2000)	$\alpha^{-0.125}$	0.875	0.875	0.875

$$\alpha = \alpha_H - 0.71(1 - \alpha_H)^{0.5} Fr^{-0.045} \left(1 - \frac{P}{P_c}\right)$$

The correlation of **Guzhov et al.** (1967) which can handle the plug and stratified flow regimes in pipes with small inclination angles to the horizontal ( $\pm 9^\circ$ ) is also considered here. The correlation is a function of the homogeneous void fraction and the mixture Froude number.

$$\alpha = 0.81\alpha_H(1 - \exp(-2.2\sqrt{Fr}))$$

The correlation **Loscher and Reinhardt** presented in 1973 as reported by Friedel(1977) is

$$\alpha = \alpha_H - \left(\frac{P}{P_c}\right)^{-0.22} \alpha_H^{1.39} (1 - \alpha_H)^{0.8} Fr^{-0.25} \left(1 - \frac{P}{P_c}\right)^{3.4}$$

**Greskovich and Cooper** (1975) developed a correlation from air water data for inclined flows. It was noted that the data showed little diameter dependency above 2.54cm but was considerably dependent on inclination angles.

$$\alpha = \frac{1}{\left[1 + 0.671 \left( \frac{(\sin \theta)^{0.263}}{Fr^{0.5}} \right) \right]} \alpha_H$$

**Moussali** correlation reported by Isbin and Biddle (1979) is given by

$$\alpha = K_{MO} \alpha_H$$

$$K_{MO} = 1 - \frac{(30.4/d_1) + 11}{60(1 + 1.6/d_1)(1 + 3.2/d_1)}$$

$$\text{Where } d_1 = \frac{1 - x}{x} \frac{\rho_G}{\rho_L}$$

Isbin and Biddle (1979) also gave the **Kutucuglu** correlation as

$$\alpha = \alpha_H - (1 - \alpha_H)^{0.5} Fr^{0.2} \left( 1 - \frac{P}{P_c} \right)^2$$

**Czop et al.** (1994) correlation was developed based on experimental measurements of void fraction in a helical tube of internal diameter 19.8mm, coil diameter of 1.17m and helix angle of 7.45° with water-SF6 mixture. It was cautioned that it may not work for situations other than for which it was developed, especially small volume flux. The correlation is given as

$$\alpha = -0.285 + 1.097\alpha_H$$

A slightly modified form of the original Armand (1946) correlation which takes into consideration the quality of the two phase mixture is given as the **Armand** and **Massina** correlation reported by Leung (2005). No limitations of any kind are reported for this correlation. The expression for the correlation is given by

$$\alpha = (0.833 + 0.167x)\alpha_H$$

### 2.1.3 Drift Flux Correlations

This type of correlations are based on the work of Zuber and Findlay (1965) where the void fraction can be predicted taking into consideration the non uniformity in flows and the difference in velocity between the two phases. This model is good for any flow regime. It has the general expression given by

$$\alpha = \frac{U_{SG}}{C_0 U_M + U_{GM}}$$

where  $C_0$  is the distribution parameter and  $U_{GM} = U_G - U_M$  is the drift velocity.

**Filimonov et al.** (1957) correlated steam-water data by the drift flux relation with the distribution parameter  $C_0$  and the drift velocity  $U_{GM}$  given as



$$U_{GM} = (0.65 - 0.0385P) \left( \frac{D_H}{0.063} \right)^{0.25} \quad \text{For } P < 12.7 \text{ MPa}$$

$$U_{GM} = (0.33 - 0.00133P) \left( \frac{D_H}{0.063} \right)^{0.25} \quad \text{For } P \geq 12.7 - 18.2 \text{ MPa}$$

with  $C_0 = 1$

Similarly, The correlation in 1959 by **Dimentiev et al.** as reported by Kataoka and Ishii(1987) is given as

$$\alpha = 1.07 j_g^{*0.8} D_H^{*-0.25} \left( \frac{\rho_G}{\rho_L - \rho_G} \right)^{-0.23} \quad \text{For } j_g^* \left( \frac{\rho_G}{\rho_L - \rho_G} \right)^{-0.5} \leq 3.7$$

$$\text{and } \alpha = 1.9 j_g^{*0.34} D_H^{*-0.25} \left( \frac{\rho_G}{\rho_L - \rho_G} \right)^{0.09} \quad \text{For } j_g^* \left( \frac{\rho_G}{\rho_L - \rho_G} \right)^{-0.5} > 3.7$$

**Wilson et al.** (1961) correlation as given by Gardner (1980) is

$$\alpha = 0.56157 F^{0.62086} D_v^{0.0917} \left( \frac{L}{D} \right)^{0.11033} \quad \text{For } F < 2$$

and

$$\alpha = 0.68728 F^{0.41541} D_v^{0.10737} \left( \frac{L}{D} \right)^{0.11033} \quad \text{For } F \geq 2$$

In an experiment done in 25.4mm (1 in) diameter vertical tube, **Nicklin et al.** (1962) produced an expression for the prediction of bubble velocity from which the void fraction could also be backed out. The constant 1.2 in the expression is said to be accurate for Reynolds numbers greater than 8000 and approximate for lesser values. The expression for the correlation is

$$\alpha = \frac{U_{SG}}{1.2 U_M + 0.35 \sqrt{gD}}$$

**Hughmark** (1965) developed another correlation for slug flow, the simple one for horizontal flow is given as

$$\alpha = \frac{U_{SG}}{1.2(U_{SL} + U_{SG})}$$

The correlation by **Gregory and Scott** (1969) in line with the work of Nicklin et al. (1962) is

$$\alpha = \frac{U_{SG}}{1.19U_M}$$

**Rouhani and Axelsson** (1970) developed two correlations for the different regions of flow boiling using the drift flux analysis of Zuber and Findlay (1965) that was compared with test data that covers a wide range of pressures, heat fluxes, sub cooling and mass velocities. The correlation is given by

$$\alpha = \frac{\frac{x}{\rho_G}}{\left[ C_0 \left( \frac{x}{\rho_G} + \frac{1-x}{\rho_L} \right) + \frac{U_{GM}}{G} \right]}$$

$$\text{where, } U_{GM} = \left( \frac{1.18}{\sqrt{\rho_L}} \right) (g \sigma (\rho_L - \rho_G))^{0.25}$$

$$\text{Rouhani I : } C_0 = 1 + 0.2(1 - x)$$

$$\text{Rouhani II : } C_0 = 1 + 0.2(1 - x)(gD)^{0.25} \left( \frac{\rho_L}{G} \right)^{0.5}$$

A variation of the Nicklin et al. (1962) correlation is given by **Bonnecaze et al.** (1971) as

$$\alpha = \frac{U_{SG}}{1.2U_M + 0.35\sqrt{gD} \left( 1 - \frac{\rho_G}{\rho_L} \right)}$$

**Mattar and Gregory** (1974) correlation is expressed as

$$\alpha = \frac{U_{SG}}{1.3U_M + 0.7}$$

**Kokal and Stanislav** (1989) correlated their air-oil experimental data in horizontal and near horizontal ( $\pm 9^\circ$ ) pipe using the drift flux relation and recommended their correlation for all flow regimes. It is given as

$$\alpha = \frac{U_{SG}}{1.2U_M + 0.345 \left[ \frac{gD(\rho_L - \rho_G)}{\rho_L} \right]^{1/2}}$$

Coddington and Macian (2002) reported and made a comparison of thirteen void fraction correlations that were developed based on the drift flux analysis of Zuber and Findlay (1965).

$$\alpha = \frac{U_{SG}}{C_0 U_M + U_{GM}}$$

Those reported as derived from rod bundle data with large distribution parameters and those with good predictions over the whole range of void fractions are reported here.

One of the oldest drift flux correlations for rod bundle systems was given by **Dix** in 1971. The parameters for this correlation as reported by Coddington and Macian (2002) are

$$C_0 = \frac{U_{SG}}{U_M} \left( 1 + \left( \frac{U_{SL}}{U_{SG}} \right)^{\left( \frac{\rho_G}{\rho_L} \right)^{0.1}} \right) \quad \text{and} \quad U_{GM} = 2.9 \left( \frac{g\sigma(\rho_L - \rho_G)}{\rho_L^2} \right)^{0.25}$$

The distribution factor and the slip velocity for the correlation of **Sun et al.** (1980) developed from tube as well as rod bundle data are given by

$$C_0 = \frac{1}{0.82 + 0.18 \frac{P}{P_C}} \quad \text{and} \quad U_{GM} = 1.41 \left( \frac{g\sigma(\rho_L - \rho_G)}{\rho_L^2} \right)^{0.25}$$

**Jowitt** developed a void fraction correlation in 1981 from rod bundle data.

The parameters for the drift flux expression as reported by Coddington and Macian (2002) are expressed as

$$C_0 = 1 + 0.796 \exp\left(-0.061 \sqrt{\frac{\rho_L}{\rho_G}}\right) \quad \text{and} \quad U_{GM} = 0.034 \left(\sqrt{\frac{\rho_L}{\rho_G}} - 1\right)$$

Coddington and Macian (2002) also gave the factors of the **Bestion** correlation which was presented in 1985 as

$$C_0 = 1 \quad \text{and} \quad U_{GM} = 0.188 \left(\frac{gD(\rho_L - \rho_G)}{\rho_G}\right)^{0.5}$$

Similarly, for the **Toshiba** correlation presented in 1989, constant values for these variables are given as

$$C_0 = 1.08 \quad \text{and} \quad U_{GM} = 0.45$$

**Inoue et al.** (1993) introduced pressure dependence for the distribution factor as well as the drift velocity terms in the drift flux expression. These are given as

$$C_0 = 6.76 \times 10^{-3} P + 1.026 \quad \text{and}$$

$$U_{GM} = \left(5.10 \times 10^{-3} m + 6.91 \times 10^{-2}\right) \times (9.42 \times 10^{-2} P^2 - 1.99P + 12.6)$$

**Maier and Coddington** (1997) undertook a least square fit to their experimental data and presented the distribution parameter and drift velocity with empirical constants as

$$C_0 = C_{MC1}P + C_{MC2} \quad \text{and} \quad U_{GM} = (v_1P^2 + v_2P + v_3)G + (v_4P^2 + v_5P + v_6)$$

The values for the constants are given in Table 2.

#### 2.1.4 General Void Fraction Correlations

Void fraction correlations which fall into this category are mostly empirical in nature though the basic underlying principles have been used to develop the correlations.

A general type correlation given in a plot format by **Flanigan** (1958) put into equation form by the AGA (American Gas Association) is considered. The correlation assumes that pipe inclination has no effect on void fraction and is only a function of the superficial gas velocity.

$$\alpha = \frac{1}{1 + 3.0637 U_{SG}^{-1.006}}$$

**Chisholm** and **Laird** (1958) correlation is given as

$$\alpha = 1 + \left[ \frac{0.8}{1 + \frac{21}{X_u} + \frac{1}{X_u^2}} \right]^{1.75}$$

A correlation was adapted by **Hoogendoorn** (1959) from correlations initially developed for vertical two-phase flows. It was reported that it is successful in correlating his experimental data for air-water and air-oil mixtures for horizontal smooth (ID 24 mm – 140mm) and rough pipes (ID 50mm) considered. Moreover, he stated that for the different velocity ranges considered, the effect of pipe diameter or liquid viscosity is negligible. The correlation has taken care of the slip velocity between the two phases and is given by

$$\frac{\alpha}{1 - \alpha} = K_{HO} \left[ U_{SG} \left( 1 - \frac{\alpha}{1 - \alpha} \frac{U_{SL}}{U_{SG}} \right) \right]^{0.85}$$

**Wallis** (1969) developed an expression that best fits the data of Lockhart - Martinelli.

The correlation is a function of the Lockhart-Martinelli variable and is given by

$$\alpha = \left[ 1 + X_{tt}^{0.8} \right]^{0.38}$$

**Neal** and **Bankoff** (1965) reported their correlation which was determined from experimental data collected varying the mass flow rates and the relative volumes of

Table 2 constants for the Maier and Coddington (1997) correlation

constant	Value
$C_{MC1}$	$2.57 \times 10^{-3}$
$C_{MC2}$	1.0062
$v_1$	$6.73 \times 10^{-7}$
$v_2$	$- 8.81 \times 10^{-5}$
$v_3$	$1.05 \times 10^{-3}$
$v_4$	$5.63 \times 10^{-3}$
$v_5$	$- 1.23 \times 10^{-1}$
$v_6$	$8.00 \times 10^{-1}$

the phases in a vertical mercury-nitrogen flow. The simplified form of the correlation looks like

$$\alpha = 1.25 \left( \frac{U_{SG}}{U_M} \right)^{1.88} \left( \frac{U_{SL}^2}{gD} \right)^{0.2}$$

**Beggs** (1972) developed correlations from experimental air water data in 1in and 1.5in diameter pipes for all inclination angles. The flow pattern is first predicted and different constants selected for the different flow regimes in horizontal flow. For all other inclination angles, the horizontal correlation is first calculated and then properly adjusted by an inclination correction factor. The horizontal void fraction correlations for the different flow regimes are given in Table 3. The void fraction at any angle of inclination for upward flow is given by

$$\frac{\alpha(\theta)}{\alpha(0)} = 1 + C \left[ \sin(1.8\theta) - \frac{1}{3} \sin^3(1.8\theta) \right]$$

The different values of  $C$  are shown in Table 3.

**Mukherjee** (1979) developed a correlation from experimental data obtained using kerosene-air and light lube oil-air mixtures in a 1.5 in ID pipe which could be inclined

up or down. Flow regime coordinates and angle of inclination were considered as the independent variables.

$$\alpha = 1 - \exp\left(C_1 + C_2 \sin \theta + C_3 \sin^2 \theta + C_4 N_L\right) \frac{N_{GV}^{C_5}}{N_{LV}^{C_6}}$$

It was reported that the regression coefficients ( $C_1$  to  $C_6$ ) which are given in Table 4 were obtained using a non linear regression programs.

**Gardner** (1980) came up with an improved correlation from pool void fraction data collected from the literature and making a comparison of five such correlations. The proposed correlation has two empirical constants one of which could take any of the proposed two values depending on the method of void fraction measurement for the pool void fraction. Both cases are considered in our analysis. The correlation is of the form

$$\frac{\alpha}{(1-\alpha)^{1/2}} = K_{GR} [F_D P^m]^n$$

With n being constant (2/3) while m could take the value of 0.16 or 0.3.

Of the five pool void fraction correlations considered in Gardner (1980) comparison, **Wilson et al.** (1961) and **Sterman** (1956) are considered here as the other correlations that predicted void fractions outside the physically realistic range are discarded.

**Tandon et al.** (1985) correlation is based on the Lockhart-Martinelli model for pressure drop and von Karman's velocity profile to represent the velocity distribution. It is developed for two phase annular flow.

$$\alpha = \left\{ 1 - 1.928 \text{Re}_L^{-0.315} [F(X_u)]^{-1} + 0.9293 \text{Re}_L^{-0.63} [F(X_u)]^{-2} \right\} \quad 50 < \text{Re}_L < 1125$$

$$\alpha = \left\{ 1 - 0.38 \text{Re}_L^{-0.088} [F(X_u)]^{-1} + 0.0361 \text{Re}_L^{-0.176} [F(X_u)]^{-2} \right\} \quad \text{Re}_L > 1125$$

Table 3 Coefficients for Beggs (1972) correlation

Flow pattern	Horizontal Void fraction	Inclination factor
Segregated	$\alpha(0) = 1 - \frac{0.98 \mathcal{N}^{0.4846}}{N_{FR}^{0.0868}}$	$C = (1 - \lambda) \ln \left[ \frac{0.011 N_{LV}^{3.539}}{\lambda^{3.768} N_{FR}^{1.614}} \right]$
Intermittent	$\alpha(0) = 1 - \frac{0.845 \mathcal{N}^{0.5351}}{N_{FR}^{0.0173}}$	$C = (1 - \lambda) \ln \left[ \frac{2.96 \lambda^{0.305} N_{FR}^{0.0978}}{N_{LV}^{0.4473}} \right]$
Distributed	$\alpha(0) = 1 - \frac{1.065 \mathcal{N}^{0.5824}}{N_{FR}^{0.609}}$	0

Table 4 Coefficients for Mukherjee (1979) correlation

Flow Direction	Flow pattern	Values of Coefficients					
		$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$
Uphill flow	All	-0.380113	0.129875	-0.119788	2.343227	0.475686	0.288657
Downhill Flow	Stratified	-1.330282	4.808139	4.171584	56.262268	0.079951	0.504887
	Other	-0.516644	0.789805	0.551627	15.519214	0.371771	0.393952



The correlation by El-Boher et al. (1988) was reported to be developed from 3400 data points collected from three different facilities i.e., steam/lead-bismuth, air/ water and steam/ mercury. The expression looks like

$$\alpha = \frac{1}{\left[ 1 + 0.27 \beta^{-0.69} (Fr)^{-0.177} \left( \frac{\mu_L}{\mu_G} \right)^{0.378} \left( \frac{Re}{We_L} \right)^{0.067} \right]}$$

$$Fr = \frac{U_{SL}^2}{gD} \quad Re = \frac{\rho_G U_{SL} D}{\mu_L} \quad We = \frac{\rho_L U_{SL} D}{c}$$

**Minami** and **Brill** (1987) developed a purely empirical general correlation from experimental air-water and air kerosene data and the data of Beggs (1972). The general correlation is given by

$$\alpha = \exp \left\{ - \left[ \frac{(\ln Z_1 + 9.21)}{8.7115} \right]^{4.3374} \right\}$$

**Kawaji et al.** (1987) correlation is proposed from experimental investigation on a large diameter high pressure steam water data. This correlation is in terms of dimensionless flow rates for small mass velocities. It was shown that the Spedding and Chen (1984) correlation has the same trend as their data which has been incorporated to cater for the data range with mixture velocity greater than 1.5 m/s. Their equation is

$$\alpha = 1.05 (j_L^*)^{1/2} \quad U_{SL} + U_{SG} < 1.5 \text{ m/s}$$

**Spedding and Spence** (1989) correlation is reported as

$$\frac{\alpha}{1-\alpha} = [0.45 + 0.08 \exp(-100(0.25 - U_{SL}^2))] \left( \frac{U_{SG}}{U_{SL}} \right)^{0.65}$$

**Hart et al.** (1989) developed a correlation for prediction of pressure drop and holdup in two phase horizontal flow for the very high void fraction range of more than 0.94.

The explicit equation is reported as,

$$\alpha = 1 - \frac{U_{SL}}{U_{SG}} \left\{ 1 + \left( 108 \frac{\rho_L}{\rho_G} \text{Re}_{SL}^{-0.726} \right)^{0.5} \right\}$$

**Huq and Loth** (1992) correlation is given by

$$\alpha = 1 - \frac{2(1-x)^2}{1 - 2x + (1 + 4x(1-x)\left(\frac{\rho_L}{\rho_G} - 1\right))^{0.5}}$$

With the aim of simplifying the iterative mechanistic model of Taitel and Dukler (1976), **Abdulmajeed** (1996) proposed an empirical correlation for the experimental data points collected. It was reported that this correlation was tested against his data (89 points) and 111 data points from the literature.

$$X_{tt} = \left[ \frac{U_{SG} \rho_G \mu_L}{U_{SL} \rho_L \mu_G} \right]^l \frac{\rho_L U_{SL}^2}{\rho_G U_{SG}^2} \quad \text{Turbulent } m=0.2, \text{ Laminar } m=1$$

$$\text{Defining } R = \ln(X_{tt})$$

For turbulent flow

$$(E_L)_{theoretical} = \exp(-0.9304919 + 0.5285852R - 9.219634 * 10^{-2} R^2 + 9.02418 * 10^{-4} R^4)$$

For laminar flow

$$(E_L)_{theoretical} = \exp \left( -1.099924 + 0.6788495R - 0.1232191 * 10^{-2} R^2 - 1.778653 * 10^{-3} R^3 + 1.626819 * 10^{-3} R^4 \right)$$

$$(E_L)_{True} = C_{ABD} (E_L)_{theoretical}$$

Where  $C_{ABD} = 0.528(U_{SG} U_{SL})^{-0.216121}$  is a correction factor

from which finally the void fraction could be easily calculated as

$$\alpha = 1 - (E_L)_{True}$$

Based on experimental data from six sources consisting of 283 points, **Gomez et al.** (2000) developed a correlation for predicting liquid holdup for slug flow for horizontal, inclined and vertical orientations. The data covers pipe diameters between 5.1 – 20.3 in and fluids considered were air, nitrogen, Freon, water and kerosene. The liquid slug is seen to be dependent on the inclination angle, mixture velocity and viscosity of the liquid phase. Surface tension has no significant effect on the holdup in comparison to the viscosity of the liquid. The equation is of the form

$$\alpha = 1 - e^{-(0.45\theta + 2.48 \times 10^{-6} Re_M)}$$

Introducing the Froude rate parameter which is the ratio of the vapor kinetic energy relative to the energy required to pump liquid from the bottom of a tube to the top of a tube and following Wallis'(1969) model, **Graham et al.** (2001) developed a correlation of the form

$$\alpha = \left(1 + \frac{1}{Ft} + \frac{1}{X_{tt}}\right)^{-0.321} \quad Ft = \left(\frac{G^2 x^3}{(1-x)\rho_G^2 g D}\right)^{0.5}$$

After listing the void fraction correlations collected from the literature into their respective categories where it is thought to give some insight into the similarity between the correlations, the next logical step one would look into is if any comparison work has been done in order to identify the “best” or a promising correlation for all or any particular area of interest. Hence, a search was made to study most if not all of the previous comparison works done along this line which we will briefly present in the next section.

## **2.2 Previous comparison work**

This section will look into void fraction correlation performance comparisons done before. It is to be noted that different investigators in the different industries in which the void fraction is an important factor to be determined have done various comparisons with selected void fraction correlations commonly used in that specific industry while some other have tried to consider a broader range of correlations that are used in different applications.

### **2.2.1 Dukler et al. (1964) Horizontal Pipe Comparison**

The first void fraction correlation comparison work was done by Dukler et al.(1964). It consists of 706 refined void fraction data points of Hoogendoorn (1959) obtained from tests run in 1,2,3 ½ and 5 ½ in diameter horizontal pipes with liquid viscosities of 3 and 20cp. The void fraction correlations considered were Hoogendoorn (1959), Hughmark (1962) and Lockhart and Martinelli (1949).

Using statistical tools such as arithmetic mean deviation, standard deviation and defining a new variable to account for the fractional deviation which includes 68% of the population to measure the spread of data, it was shown that the Hughmark (1962) correlation was able to perform better than the other two.

### **2.2.2 Marcano (1973) Horizontal Pipe Comparison**

Marcano (1973) compared five correlations; Lockhart and Martinelli(1949), Hughmark(1962), Dukler et al. (1969), Eaton et al. (1967),Guzhov et al. (1967) and Beggs(1972) correlations using the data of Eaton (1966) and Beggs(1972).

The total data points consisted of 238 natural gas - water (Eaton) and 58 air - water (Beggs). The statistical parameters considered in the Dukler et al. (1969) comparison were calculated here to determine the relative accuracy of the correlations. It was shown that the Eaton et al. (1967) and Beggs (1972) correlations performed well

owing to the fact that the data used for comparison is the data from which these correlations were developed. The correlations of Dukler et al. (1969) and Lockhart and Martinelli (1949) are reported to have acceptable results while the rest were totally unsatisfactory.

Dropping out the “unreliable” low liquid holdup data (below 0.1), the correlations were further compared. It was noted that the performance of the correlations was better. The performance of the correlations for specific liquid holdup(void fraction) ranges were also analyzed where Eaton et al. (1967), Guzhov et al. (1967) and Beggs(1972) correlations were found to do better for void fraction less than 0.65(or liquid holdup above 0.35) while on the lower range of holdup only the Eaton et al. (1967) correlation gave acceptable results. For liquid holdup below 0.1 none of the correlations gave reasonable accuracy though the Dukler et al. (1969) correlation and the no slip model gave best estimates. One of the many reasons for this could be the unreliability of the measurement of the data for this range of holdup.

### **2.2.3 Palmer (1975) Inclined Pipe Comparison**

Using 174 liquid holdup(void fraction) water-natural gas experimental data from a 2 in diameter pipe line with three uphill(at 4.2,7.1 and 7.5 degrees from the horizontal) and three downhill(at 4.3,3.8 and 6.3 degrees from the horizontal) test sections, Palmer(1975) compared the correlations of Beggs(1972), Flanigan (1958) and Guzhov et al. (1967). The percent error, average percent error and the standard deviation were calculated to make the comparison. It was concluded that the Beggs(1972) correlation gave good predictions of the void fraction for uphill flow. The Flanigan (1958) correlation was the least accurate which may be expected as it didn't consider downhill flow and also the fact that it was given as an approximate estimate by Flanigan.

#### **2.2.4 Mandhane et al. (1975) Horizontal Pipe Comparison**

Mandhane et al. used 2700 void fraction (liquid holdup) data contained in the University of Calgary Multiphase Pipe Flow Data Bank. A two step procedure for calculating the void fraction was used in that the flow pattern is first predicted and then a correlation developed for that flow regime is used to calculate the liquid holdup. The flow pattern map of Mandhane et al. (1974) was used for the analysis. Twelve void fraction correlations were considered. They were, Lockhart and Martinelli(1949), Hoogendoorn(1959), Eaton et al. (1967), Hughmark(1962), Guzhov et al. (1967), Chawla(1969), Beggs(1972), Dukler et al. (1969), Scott(1962),Agrawal et al.(1973), Hughmark(1965) and Levy(1960).

Five different measures of error, namely the root mean-square error, mean absolute error, simple mean error, mean-percentage absolute error and the mean-percentage error were used. Arbitrary designation of the void fraction ranges to the flow pattern was made to see the predictive capability of the correlations within each range of holdup (void fraction) values or flow regime.

Using the flow pattern map of Mandhane et al.(1974), the Hughmark(1962) correlation is recommended for the bubble, elongated bubble and slug flow regimes, the flow pattern specific correlation of Agrawal et al.(1973) for the stratified, Lockhart and Martinelli(1949) correlation for annular, annular-mist and the Beggs(1972) correlation for the dispersed-bubble flow regimes. No correlation was recommended for the total data points even though the recommended error values for all the parameters were given.

Comparing the correlation using the flow maps of Hoogendoorn(1959), Govier and Aziz(1972) and Baker(1954), it was shown that the Hughmark(1962) correlation

predicts the bubble, elongated bubble regimes in all the four maps and the slug regime in all but the Baker (1954) map where the Chawla(1969) is seen to outperform it.

The stratified regime is predicted by the Agrawal et al. (1973) correlation in the Mandhane et al. (1974) and Baker (1954) maps while the Dukler et al. (1964) correlation predicts that of Hoogendoorn (1959) and Govier and Aziz (1972). This is easily explainable as it has an advantage over the others due to the bias it has towards these two data sets from which it was developed.

The annular, annular-mist regime is handled by the Lockhart and Martinelli (1949) in all the flow pattern maps.

In the dispersed-bubble regime the Beggs (1972) correlation has predicted the data in Mandhane et al. (1975) while Hughmark (1962) was able to handle that of Hoogendoorn (1959) and Govier and Aziz (1972). The Hoogendoorn (1959) correlation is shown to predict the data in the Baker (1954) map quite well.

All in all it was concluded that none of the correlations gave satisfactory results in the annular and annular mist flow regimes.

#### **2.2.5 Papathanassiou (1983) Horizontal Pipe Comparison**

Deducing the physically realistic range within which two phase flow exists for a set of operating conditions and specified fluid, a void fraction spectrum graph is constructed which is used to compare the correlation of Lockhart and Martinelli (1949), Hoogendoorn (1959), Bankoff (1960) and Hughmark (1962). All but the Bankoff (1960) correlation have similar trend in predicting void fraction. The Bankoff (1960) correlation is seen to under predict the void fraction mainly due to the fact that it was developed for vertical flow. It was shown that the Lockhart and Martinelli (1949) correlation falls out of the physically realistic range at a void fraction of about 0.4. At very high void fraction ranges of 0.7 and above the Bankoff (1960) correlation is seen

to fall in the unrealistic region. The disagreement between the void fraction correlation predictions in the lower and upper extreme values of void fraction ranges is explained by the fact that few experimental results at this range exist when the correlations were developed.

#### **2.2.6 Spedding et al. (1990) Inclined (2.75°) Pipe Comparison**

Spedding et al. (1990) considered 60 correlations and the data of Spedding and Nguyen (1976) to make a comparison of the void fraction correlation at an upward angle of 2.75° from the horizontal. A recommendation for the 12 flow regimes in this flow was made. Criteria that the average predicted value to be within  $\pm 15\%$  with a 30% spread was used to consider the predictive performance of a correlation as satisfactory.

#### **2.2.7 Abdulmajeed (1996) Horizontal Pipe Comparison**

In an effort to simplify the mechanistic model of Taitel and Dukler (1976) and developing a new correlation, Abdulmajeed (1996) collected 88 air-kerosene void fraction data in a horizontal 2in diameter pipe and compared 12 correlations. Namely; Armand (1946), Hughmark and Pressburg (1961), Hughmark (1962), Eaton et al. (1967), Guzhov et al. (1967), Beggs (1972), Gregory et al. (1978), Brill et al. (1981), Chen and Spedding (1981, 1983), Mukherjee and Brill (1983), Minami and Brill (1987) and Abdulmajeed (1996). The comparison parameters used were average percent error, absolute average percent error and the standard deviation. It was shown that the correlation of Abdulmajeed (1996) is able to predict void fraction in stratified, slug and annular flow regime which covers a range of flow regimes in contrast to the implicit Taitel and Dukler (1976) model which was specifically developed for stratified flow.



### **2.2.8 Spedding (1997) General (-90° to +90°) Comparison**

Spedding undertook an extensive comparison on more than 100 void fraction correlations using the air-water data of Spedding and his coworkers (1976, 1979, 1989, 1991, 1993). The pipe diameters ranged from 26mm to 95.3 mm. Prediction of a correlation was considered satisfactory for average predicted values that fall within  $\pm 15\%$  of the data with spread of individual values within  $\pm 30\%$ . Eighteen flow regimes were identified for which the predictive capability of the correlations assessed. The comparison ranges from  $-90^\circ$  to  $+90^\circ$  with fair representation of major inclination angles.

It was noted that no single model could handle all flow regimes and angle of inclination satisfactorily. Different void fraction correlations are recommended for different flow regimes for horizontal/upward inclined and downward flow separately.

### **2.2.9 Friedel and Diener (1998) Horizontal/Vertical Upward Comparison**

Based on 24000 experimental data bank in single component water and R12, and two component air-water, a comparison of 13 (refined selection of the originally 26 correlations) void fraction correlations was undertaken. Some of the correlations are proprietary of which no information was given while the others are collected from the open literature. Some of the correlations considered in the analysis that passed the limiting void fraction criteria, for  $x = 0, \alpha = 0$  and  $x = 1, \alpha = 1$ , and were considered in our comparison are Huq and Loth (1992), Kowalczewski (1964), Smith (1969), the Loscher and Reinhardt correlation reported by Friedel (1977), Rouhani and Axelsson (1970) and Premoli et al. (1970).

The average predictive accuracy of the correlations was established by calculating the statistical parameters; the scatter of logarithmic ratios, scatter of absolute deviations and average of logarithmic ratios for the mean void fraction and mean density.

The Rouhani and Axelsson's (1970) first correlation (Rouhani I) has been recommended as having an accurate predictive capability.

Coming to the conclusion of this chapter, the challenge of the design engineer in finding a suitable correlation that would be applicable to the task at hand from the numerous correlations available is unquestionable. Moreover, the few numbers of void fraction correlation comparisons made and the recommendations put forward by the investigators are very narrow in nature complicating the task of reaching at objective conclusions very difficult. This is due to the fact that, the correlations selected for comparison are those that are popular to a particular industry (like the petroleum, nuclear industries and so on), limited to a certain physical restriction which may not be always practical (horizontal pipe, air –water mixture) or biased towards one experimental set up for which complete and detailed description of and/or the data that was obtained thereby are not clearly indicated.

Hence, an objective assessment of all the available correlations in the open literature with all the available data from different sources covering wide range of operating characteristics including any recent data which was not included in the previous comparisons is justified. The next section of this work is devoted to the presentation of the systematically gathered and compiled, acceptably good quality void fraction database.

## CHAPTER 3

### EXPERIMENTAL DATA BASE AND FLUID PROPERTIES

As pointed out in the previous chapter, the main emphasis of this section would be on the characteristics of the experimental data base that was collected from the literature and the various correlations used in determining fluid physical properties of the test mixtures. A brief highlight to the different void fraction measurement techniques used by different investigators would be given towards the end of this chapter.

In the collection process, effort has been made to make the data base as un-biased, wide range covering and having an acceptable quality as was practically possible.

#### **3.1 Experimental Database**

After the initial screening process that primarily considered the completeness of the data set, the data of Eaton (1966), Beggs (1972), Spedding and Nguyen (1976) , Mukherjee (1979), Minami and Brill (1987), Franca and Lahey (1992), Abdulmajeed (1996), and Sujumong (1997) were selected with which the performance of the void fraction correlations are to be compared. The range of diameters, inclination angles and fluids covered by the data is summarized in Table 5. A more detailed description of the each data set is provided in Appendix A.

The pressure, temperature, diameter of the pipe and the measured and calculated values for the void fraction from each source is taken as it is, without changing any of the units involved. One exception is that of the Spedding and Nguyen (1976) data, where a value for the ratio between the void fraction to the liquid holdup was reported from which the void fraction is explicitly calculated.

Each set of data was tested against simple but essential requirements to determine if all the data points are within the realistic range of values.

One screening criterion was to identify the measured void fraction in any two phase flow that exceeded the value calculated by the homogeneous model. This yielded 13 of Beggs (1972), 20 of Spedding and Nguyen (1976), 61 of Mukherjee (1979), 1 of Eaton (1966), 5 of Abdulmajeed (1996), 3 of Franca and Lahey (1992) and 3 of Sujumnong (1997) data to be out of the realistic range. These data points were therefore excluded from the data bank.

In addition, following the work of Zuber and Findlay (1965), plotting any two phase flow experimental data on the weighted mean velocity ( $U_{SG}/a$ ) versus average volumetric flux density ( $U_M$ ), a few abnormally isolated data points that don't fall into any data clusters (accounting for flow pattern change) were suspect to be unreliable and hence had been taken out. As pointed out by Zuber and Findlay (1965), deviation of points from a certain cluster may be attributed to a change in flow regime.

When a group of data points stray far from the rest, verification with the reported flow pattern by the researcher has been done to check if indeed those data points belong to a different flow regime. If not, these data points are suspect to be unreliable and hence discarded.

Data points which are identical in all aspects(repeated runs) but reported as separate points in some of the reports were also taken out. This is to make the analysis fair especially for those correlations that could not predict that data point as calculating the total percentage of data points within the acceptable percentage band is sensitive to the number of data points and counting the same data point more than once is not necessary.

Table 5 Characteristics of Data Base Sources

Source	Physical flow configuration / Characteristics	Mixture considered	Measurement Technique	No. of Data points
Eaton(1966)	Horizontal, ID 52.5mm and 102.26mm	Natural Gas-Water	Quick-Closing Valves	237
Beggs(1972)	Horizontal, Uphill and Vertical ID 25.4mm and 38.1mm	Air-Water	Quick-Closing Valves	291
Spedding and Nguyen(1976)	Horizontal, Uphill and Vertical ID 45.5mm	Air-Water	Quick-Closing Valves	1383
Mukherjee(1979)	Horizontal, Uphill and Vertical ID 38.1 mm	Air-Kerosene	Capacitance Probes	558
Minami and Brill(1987)	Horizontal, ID 77.93mm	Air-water and Air-Kerosene	Quick-Closing Valves	111
Franca and Lahey(1992)	Horizontal, ID 19mm	Air-Water	Quick-Closing Valves	81
Abdulmajeed(1996)	Horizontal, ID 50.8mm	Air-Kerosene	Quick-Closing Valves	83
Sujumnong (1997)	Vertical, ID 12.7mm	Air-Water	Quick-Closing Valves	101

Note: For details on this data set, please refer to Appendix A.

The next step taken was to collect well known correlations and equations that could accurately predict the fluid physical properties of the fluids under consideration. We shall separately discuss each of the fluids considered as the types of correlations developed differ from one type of fluid to the other.

### **3.2 Fluid Physical Properties**

The experimental data base compiled from the different sources consists of mixtures of four fluids. These are air, kerosene, natural gas and water. Properties for these fluids were calculated with correlations either explicitly given by the researcher of the specific data or from well known correlations and equations of state taken from the literature. We will briefly give the correlations used for each fluid type in this section.

#### **3.2.1 Properties of Air**

Air being the most economical fluid available for any experiment, it has been used as the gas phase in all data sets except that of Eaton(1966). Neglecting  $Z$ , compressibility effects, the density of air is calculated from the ideal gas equation of state as

$$\rho_{air} = \frac{P}{T(R/M_{air})} \quad \text{in (kg/m}^3\text{)}$$

Where  $T$       Temperature in K

$P$       Absolute pressure in Pa

$R$       The universal gas constant (8314.34 J/kmol. K)

$M_{air}$       Molecular weight of air (28.966 kg/kmol)

The viscosity of air is calculated from the correlations by **Kays and Crawford**(1993) which is reported to be good for the temperature ranges between -10 to 120 °C with  $R^2 = 0.99994$ . It is given as

$$\mu_{air} = 1.7211 \times 10^{-5} + 4.8837 \times 10^{-8} T - 2.9967 \times 10^{-11} T^2 \quad \text{in (Pa.s)}$$

where  $T$  is in  $^{\circ}C$ .

### 3.2.2 Properties of Kerosene

Mukherjee (1979), Minami and Brill (1987) and Abdulmajeed (1996) have used kerosene as the liquid phase in their experiments. Properties such as density, dynamic viscosity and surface tension for kerosene were given by approximate equations based on laboratory measurements. The respective equations put forward by each researcher have been used in determining the physical properties of kerosene for the respective data set.

The fitted equations by **Mukherjee** (1979) are given as

$$c_{\text{ker}o} = 29.198 - 0.05T \quad \text{in (dynes/cm)}$$

$$\rho_{\text{ker}o} = 52.8858 - 0.0289T \quad \text{in (lb/ft}^3\text{)}$$

$$\mu_{\text{ker}o} = \text{Exp}(1.4344 - 0.0115T) \quad \text{in (cp)}$$

where  $T$  is in  $^{\circ}F$ .

The fitted equations by **Minami and Brill** (1987) are

$$c_{\text{ker}o} = 33.62 - 0.06452T \quad \text{in (dynes/cm)}$$

$$\rho_{\text{ker}o} = 62.4(0.83756 - 0.000395T) \quad \text{in (lb/ft}^3\text{)}$$

$$\mu_{\text{ker}o} = 4.955\text{Exp}(-0.0111T) \quad \text{in (cp)}$$

where  $T$  is in  $^{\circ}F$ .

The fitted equations by **Abdulmajeed** (1996) are given as

$$c_{\text{ker}o} = 29.9776 - 0.13176T \quad \text{in (dynes/cm)}$$

$$\rho_{\text{ker}o} = 62.4(0.8252 - 0.00072T) \quad \text{in (lb/ft}^3\text{)}$$

$$\mu_{\text{ker}o} = 3.473004\text{Exp}(-0.02016T) \quad \text{in (cp)}$$

where  $T$  is in  $^{\circ}C$ .

### 3.2.3 Properties of Natural Gas

The ideal gas equation of state accounting for compressibility effects has been used in calculating the density of natural gas. Hence, the density of natural gas is calculated from

$$\rho_{Ngas} = \frac{P}{ZT(R/M_{Ngas})} \quad \text{in (kg/m}^3\text{)}$$

The molecular weight of the natural gas used in the Eaton (1966) experiment is calculated from the specific gravity of the natural gas given and the known molecular weight of air.

The compressibility factor,  $Z$ , is calculated by a FORTRAN subroutine of **Dranchuk and Abou-Kassem** (1975) which is the approximation of the **Standing and Katz** (1942) correlation. As inputs to the subroutine, the pseudo-critical properties, pressure and temperature have to be determined first.

Since the composition of the natural gas used in Eaton (1966) experiment is not given, the pseudo-critical properties could only be determined from the **Standing** (1977) equation which is a function of the specific gravity of the natural gas only. The equations for the natural gas systems are given as,

$$T_{pc} = 168 + 325\gamma_{Ngas} - 12.5\gamma_{Ngas}^2$$

$$P_{pc} = 677 + 15\gamma_{Ngas} - 37.5\gamma_{Ngas}^2$$

where  $\gamma_{Ngas}$  is specific gravity of natural gas.

Then the reduced pressure and temperature for each data point are calculated from the expressions,

$$T_r = \frac{T}{T_{pc}}$$



$$P_r = \frac{P}{P_{pc}}$$

These reduced pressures and temperatures are then read from file to calculate the compressibility factor for each data point. The other thing worth mentioning here would be, Eaton(1966) reported the volume flow rate of natural gas in standard units (scf/d). Therefore, to convert this into the common volume flow rate units in m<sup>3</sup>/sec or ft<sup>3</sup>/s, the gas formation volume factor has to be considered in addition to the basic unit conversions.

The gas formation volume factor is defined as the ratio of the actual volume of gas at given pressure and temperature to the volume of the same gas at standard conditions (60°F and 14.7 Psia). **Ahmad** (2001) gives the gas formation volume factor as,

$$B_{gas} = 0.02827 \frac{ZT}{P}$$

where  $B_{gas}$  is the gas formation volume factor, ft<sup>3</sup>/scf and  $T$  is Temperature in °R.

The optimized **Lee et al.** correlation reported by **Jeje and Mattar** (2004) is used to calculate the viscosity of natural gas. The assumption made here is that the natural gas used is a sweet gas (containing no or only traces of non-hydrocarbon gases) due to its low specific gravity. The Lee et al. correlation is given by,

$$\mu_{Ngas} = 10^{-4} K \text{Exp}(X\rho^Y)$$

where the different variables are defined as,

$$X = x_1 + \frac{x_2}{T} + x_3M$$

$$Y = y_1 - y_2X$$

$$K = \frac{(k_1 + k_2M)T^{k_3}}{k_4 + k_5M + T}$$

The optimized constants for this equation as given by Jeje and Mattar (2004) are given in Table 6.

### 3.2.4 Properties of Water

As in the case of air, being the cheapest and very easy to handle, most experiments are done with water as the liquid phase. There are quite a number of correlations to determine the thermo physical properties of water in the literature. The equations by **Linstrom and Mallard** (2003) reported to be applicable to temperature ranges between 0 and 100 °C with  $R^2=0.99997$  are used to calculate the density and viscosity of water. The equations are given as

$$\rho_{water} = 999.96 + 1.7158 \times 10^{-2} T - 5.8699 \times 10^{-3} T^2 + 1.5487 \times 10^{-5} T^3 \quad \text{in} \\ (\text{kg/m}^3)$$

where,  $T$  is in °C.

$$\mu_{water} = 1.7888 \times 10^{-3} - 5.9458 \times 10^{-5} T + 1.3096 \times 10^{-6} T^2 - 1.8035 \times 10^{-8} T^3 \\ + 1.3446 \times 10^{-10} T^4 - 4.0698 \times 10^{-13} T^5 \quad \text{in(Pa.s)}$$

where,  $T$  is in °C.

**Popiel and Wojtkowiak** (1998) gave a formula for calculating the surface tension of water that approximates the data of **Vargaftik et al.** (1983,1996) for the temperature ranges between 0 and 150 °C as,

$$\sigma_{water} = 0.075652711 - 0.00013936956 T - 3.0842103 \times 10^{-7} T^2 + 2.7588435 \times 10^{-10} T^3$$

where,  $T$  is in °C and  $\sigma_{water}$  in (N/m).

### 3.2.5 Measurement Techniques

As pointed out in the beginning of this chapter, a brief description of some of the most commonly used experimental void fraction methods would be discussed. It must be pointed out that the purpose of this short section here is to bring to the attention of the

Table 6 Optimized constants for the Lee et al. correlation

Variable	Optimized Value
$k_1$	16.7175
$k_2$	0.0419188
$k_3$	1.40256
$k_4$	212.209
$k_5$	18.1349
$x_1$	2.12574
$x_2$	2063.71
$x_3$	0.00119260
$y_1$	1.09809
$y_2$	-0.0392851

interested reader what the possibilities are in terms of measuring void fraction and some of the pros and cons associated with some of the commonly used methods. It also presents some of the very good references which should be looked into if in depth understanding in this area is sought.

A considerable amount time and investment has gone into researching and devising a proper measurement technique that could capture void fraction data accurately. As the applications and areas of interest to the different researchers were diverse, the number of void fraction measurement devices or techniques also differ in various aspects ranging from but not limited to cost, nature of data of interest (local and/or average) and simplicity in construction as well as operation.

Hewitt (1978) discussed the various measurement techniques available to measure void fraction classifying them into different categories. This section would be predominantly a take off from his discussion with emphasis given only to those which could be commonly encountered in the literature.

Three of the most common of the available experimental techniques given by Hewitt (1978) are the radioactive absorption and scattering method, impedance method and the direct and indirect volume measurement. A brief discussion and the relative advantages and disadvantages of these methods will be presented below.

### **Radioactive Absorption and Scattering Technique**

As the name implies this method relies on the property of the two phase fluid mixture to attenuate or scatter a given beam of radioactive ray.  $\beta$  (beta), Gamma and X rays have been used by different investigators while the most commonly employed one being the Gamma ray attenuation technique. This technique will give the instantaneous local void fraction which may also be integrated over time to calculate an average while the collected data could simultaneously give prediction on the flow pattern. One of the major advantages with this method is that it doesn't disrupt the flow field while the safety consideration, high cost, complexity of construction and inaccuracy in measurement are some of the drawbacks associated with this method. A traversing mechanism that scans the length of the test section is a peculiar feature of this technique.

### **Impedance Technique**

Incorporating the appealing nature of being non intrusive, continuous measurement, less expensive and simple construction, this technique has been used more commonly to measure void fraction. This method takes advantage of the change in electrical impedance between two electrodes separated by a dielectric material, for this case the dielectric material is the two phase mixture and the change in impedance would be triggered as a result of the variation in void fraction. Designs for the electrodes can be flush mounted to the inner wall of the test section or mounted in a different pattern around the external body.

A number of devices based on either the conductance or capacitance or both have been developed. Some of these are the works by Gregory and Mattar (1973), Rosehart et al. (1975), Mukherjee (1979), Elkow and Rezkallah (1996) and Song et al. (1998).

The draw backs of this method are the temperature dependency of the dielectric constant, requirement of high excitation frequency for large conductivity fluid and also calibration issue associated with purity of the mixture.

### **Direct Measurement Technique**

This is the most commonly used experimental technique where the two phase flow is trapped between two quick closing valves mounted along the test section after which the trapped volume of liquid is drained and measured. Since the total volume of the test section between the valves is predetermined by design, the ratio of the measured amount to that of the total would then give the required fraction of the liquid and hence indirectly determining void fraction.

The most noticeable advantages of this method are simple construction and low cost. It is also used to calibrate and/or validate void fraction measurements from other methods. The unique feature of this method is that a bypass line would be required not to damage the system due to disruption to the flow. The major set backs in using this system are that it is time consuming, would not capture any transient characteristics of flow and not practicable to high temperature and pressure applications.

For someone thinking of building up or putting together a system for the purpose of measuring void fraction, it is recommended to start with Hewitt (1978) where he has discussed some other methods on top of the ones presented above while presenting an appreciable number of references which could be of practical interest.

To wrap up this chapter, we have briefly presented the different data sets we included in our database. Applying some basic screening tools to make the data sets more

reliable we have excluded data that do not fall within the physically realistic band within which two phase flow should exist. Well known and widely used equations and correlations for calculating the physical properties of the fluids under consideration have been gathered from primary and secondary sources and reported in this section. Having highlighted the shortcomings of the previous performance comparison work that were focused on a few selected correlations or limited data sets, we have tried to gather a good number of void fraction correlations (Chapter 2) and reasonably wide ranging experimental void fraction data (this chapter). We are now ready to do a comprehensive comparison of all the void fraction correlations with the experimental data base that was compiled.

The next chapter will be focused on reporting the methodology and results of the comparison work that was undertaken.

## **CHAPTER 4**

### **EVALUATION OF VOID FRACTION CORRELATIONS**

This chapter is mainly concerned with the evaluation of the predictive capabilities of the different void fraction correlations that were reported in Chapter 2 with the data base discussed in Chapter 3.

For ease of reference and quick read-through between some of the similarities and/or disparity of the conclusions drawn from this work and the previous assessments, the current comparison work is divided into three sections namely horizontal, upward inclined and vertical. These sections will be separately analyzed and a recommendation drawn for each section before the final summary is presented.

In line with our ambitious goal of recommending a single or a combination of correlations that could handle all angles regardless of flow regimes, inclination angles, fluid types and so on, we have removed all the restrictions on all the correlations in trying to see their promise to handle most if not all of the diverse data points in the database.

The fact that this is an over stretch on all of the correlations should be well noted. Hence, all subsequent analyses and comparisons made which may dictate unfavorable conclusions on some or all correlations should be taken from this perspective only and should not be an undermining factor against the correlations in any way.

The degree of accuracy expected from a correlation for a particular application is a very subjective issue recognizing the fact that none of the correlations would handle all data points in any data set. Hence, the practicing engineer/researcher could set his

own criteria of choosing the “best” performing correlation considering the task at hand.

As given in the literature review part, different parameters have been used by the previous comparison analyses to show the relative accuracy of one correlation over the others. The most commonly used of these include widely known parameters like percent error and standard deviation while some investigators have defined other parameters. The fractional deviation above and below the mean used by Dukler et al. (1964), the scatter and average of logarithmic ratios by Friedel and Diener (1998) and the relative performance factor reported by Abdulmajeed and Al-Mashat (2000) are noteworthy.

All of the methods have inherent strengths and weaknesses. Preference of one technique over the other is again a subjective matter. We have considered the simple percentage error index in this work for two reasons. The first one is that we would get hold of some very “stray” data points that would have gone undetected due to the cancellation effect introduced in some of the error measurement techniques.

The other rationale is that it would give us an insight as to where the weakness of a correlation lies which could facilitate the investigation to improve a given correlation or to learn from the pitfalls when developing a new one.

The major drawback of this technique is that there is no general consensus on what a “good” bandwidth is for a given correlation to be taken as acceptably accurate. Different researchers have considered different ranges for their comparisons. Madsen (1975) took  $\pm 10\%$  for checking his correlation against measured data; Chexal et al. (1991) considered  $\pm 10\%$  when assessing eight void fraction correlations that are common in the nuclear industry. Smith (1969) also took  $\pm 10\%$  for verification of his correlation against experimental data and also assessing the relative importance with



other correlations. Spedding et al. (1990) and Spedding (1997) have considered the  $\pm 15\%$  index together with a maximum spread of 30%. Friedel and Diener (1998) have presented prediction of two correlations on  $\pm 30\%$  index as an example even though the final recommendations were not based solely on that plot.

The other major area that should be looked into when comparing a given correlation with a particular data set is the total number (percentage) of the data points that have to be correctly predicted within the specified index to deem the correlation good or not. We would refer to this percentage as the cutoff percentage not to confuse with the percentage error index discussed above.

In order to be able to cater for different (at least three) scenarios to make comparisons between the correlations that one could pick from; a decision was made to go for a progressively increasing cutoff percentage as the percentage error index is relaxed out. By this we mean that a lower cut off percentage of 75% was chosen for the more restrictive 5% error index while cutoff percentages of 80 and 85% were used for the 10 and 15% indices respectively.

Each of the data sets, even those from the same researcher but with different inclination angle or fluid, has been considered separately. This would aid in making sound judgment about the data, the capability of the correlations to predict that specific data set and other pertinent facts that could have been easily obscured if two or more data sets were bundled together.

Some approximations and modifications were done on some correlations without compromising their capability so that the calculations could be done more efficiently. A brief explanation of the two major such adjustments are given below.

The correlation by Greskovich & Cooper (1975) was developed for inclined flows only and hence could not directly be applied to horizontal flow case. Moreover,

setting a value of zero for the inclination angle in the correlation for horizontal flow, the different data sets tested were not predicted well. However, the results were seen to significantly improve when a very small number near zero ( $10^{-6}$ ) is considered and this has been integrated in the calculations.

The Hughmark (1962) correlation flow factor  $K_{HU}$  which was given in tabular form in the original report in terms of the variables discussed in Chapter 2, has been approximated by a function fitted to the values given with a correlation coefficient of  $R^2=0.999$ .

For all correlations requiring iteration, the simple but accurate bisection method has been used to calculate the void fraction that has lower and upper limits of 0 and 1 respectively.

We shall now present the different comparisons starting with the horizontal flow first.

#### **4.1 Horizontal flow comparison**

A total of 900 data points from eight sources was used for the comparison in this section. The analysis is first focused on the predictive capability of the correlations on each of the data sets separately and would be consolidated into a general one towards the end of the section.

The total number of data points correctly predicted by each of the correlations for the respective individual data sets are reported in Table 7. This table is the base on which all subsequent percentage tables and analyses there of are made. It gives the reader an objective assessment ground of the capability of each correlation between the different data sets as percentage values could sometimes give a misleading impression at first glance.

To make the relative comparison between the correlations, the numbers of correctly predicted points are expressed as a percentage for the different error percentage

indices. These have been presented in Table 8 with a highlight on those results which have met our criteria set out earlier and are worth noting.

The data set of Eaton (1966) could not be satisfactorily predicted by any of the correlations for the more restrictive 5% index. The closest best prediction is given by Premoli et al. (1970) that could only predict 68.4% of the total data points.

The same trend is observed for the data of Abdulmajeed (1996) and Mukherjee (1979). The only exception are the correlations of Filimonov et al. (1957) and Premoli et al. (1970) that were able to predict 77.1% and 74.7 % of Abdulmajeed (1996) data. The next best prediction comes from Mukherjee (1979) and Minami and Brill (1987) with 72.3% and 73.5%, respectively of the data points correctly predicted for the Abdulmajeed (1996) data. The performance of all the rest of the correlations was far from being satisfactory. Even those of Mukherjee (1979) and Minami and Brill (1987) failed to give acceptable prediction of Mukherjee (1979) data.

This clearly shows the lack of capability of all of the correlations to satisfactorily predict void fraction for fluids different from air-water mixtures from which most of them were originally developed. A point worth mentioning here is that the correlations of Mukherjee (1979) and Minami and Brill (1987) have an edge over the others as they are developed from the same data set that was used here for the comparison.

The other notable observation here is that even the correlations developed from the respective data sets by Mukherjee (1979) and Abdulmajeed (1996) could only capture 51.6% and 57.8% of their own data within the 5% percentage error index. The exception in this regard is the correlation by Minami and Brill (1987) that was able to predict 84.2% of their own air kerosene data within the 5% index. However, it captured a fair 73.5% of Abdulmajeed (1996), with only 53.2% of Mukherjee (1979)

and 55.3% of Eaton (1966) data. Hence, even for the same or similar fluid types, correlations developed from specific experimental data sets with fitted constants fail to adequately predict data sets from other test setups and operating conditions.

The trend in predictive capability for the air-water data is almost similar for the 5% index with a few exceptions. The Beggs (1972) correlation was able to predict 82.1 % of his own data while Lockhart and Martinelli (1949), Wallis (1969) and Spedding and Chen (1984) predicted 74.1, 74.4 and 75.6% of Spedding and Nguyen (1976) data, respectively.

The performance of all correlations improves as the percentage error index is relaxed out to 10 and 15%. With the 10% error percentage index 9, 7 and 9 correlations were able to predict the data of Abdulmajeed(1996), Eaton(1966) and the air kerosene data of Minami and Brill(1987), respectively. It must be noted here that only the correlation of Premoli et al. (1970), Mukherjee (1979), Minami and Brill(1987) and Graham et al. (2001) were able to predict two data sets within this index whereas all the rest only predicted a single data set.

Considering all the data sets, the correlations of Armand-Massina (Leung 2005), Premoli et al. (1970) and Minami and Brill (1987) are the ones which came close to the set criteria and worth a general recommendation for the 10% percentage error index.

The performance of most of the correlations further improves with an extended error percentage index of 15%. Here a fair number of correlations were seen to perform rather well as compared to the lower error percentage indices.

The ones worth consideration here are due to Armand-Massina as reported by Leung (2005), Hughmark (1962), Smith (1969), Premoli et al.(1970), Rouhani and Axelsson (1970)first Correlation (Rouhani I), Dix(1971), Beggs(1972), Chisholm(1973),

Mukherjee(1979),Chisholm(1983)-Armand(1946),Toshiba(1989),Huq & Loth(1992), Minami & Brill(1987) and Graham et al.(2001).

Consolidating the results of the individual data sets together, a comparison table was constructed that shows the total number and percentage of data points correctly predicted for the whole horizontal database (900 points). This has been presented in Table 9.

As this is a cumulative analysis, all the issues we raised when discussing the individual data sets would have their own effect on the overall performance of each correlation. This is reflected in the table as the performance of all the correlations have deteriorated heavily especially within the 5% error index. The only correlation that was able to give a fair result is that of Premoli et al. (1970) with 65.2% which is much lower than the “acceptable” cutoff index we set out earlier.

The case is seen to improve for the wider error indices where the correlation of Armand-Massina (Leung 2005), Minami and Brill (1987), Premoli et al. (1970) and Rouhani I (1970) were able to predict 79.1, 78.3, 78.2 and 77 % of the whole data set within 10% error index, respectively. On the other hand correlations that gave an acceptably good prediction within the 15% index are Rouhani I(1970), Hughmark (1962), Armand Massina(Leung 2005), Mukherjee(1979), Minami and Brill(1987), Chisholm(1983)-Armand(1946) and Chisholm(1973) in order of decreasing accuracy. Another set of correlations worth a general recommendation with very close performance to the ones listed above are due to Smith (1969), Premoli et al. (1970), Dix (1971),Beggs (1972),Toshiba(1989), Huq and Loth (1992) and Graham et al. (2001).

Table 7. Number of data points correctly predicted for horizontal flow by data sets

Correlation / Data Source	Eaton (1966) (Natural Gas - Water)			Beggs (1972) (Air - Water)			Spedding & Nguyen (1976) (Air - Water)			Mukherjee (1979) (Air - Kerosene)			Minami & Brill (1987) (Air - Water)			Minami & Brill (1987) (Air - Kerosene)			Franca & Lahey (1992) (Air - Water)			Abdulmajeed (1996) (Air - Kerosene)		
	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%
Percentage within	Total Points 237			Total Points 56			Total Points 270			Total Points 62			Total Points 54			Total Points 57			Total Points 81			Total Points 83		
Homogeneous	58	104	146	14	22	35	108	143	166	19	39	47	12	21	28	14	26	33	8	27	49	34	52	62
Armand(1946)	53	116	185	23	36	50	41	90	161	10	23	52	15	27	42	14	30	47	29	56	73	5	23	54
Armand - Massina <sup>1</sup>	118	187	205	39	50	54	135	221	236	21	44	55	33	42	45	32	44	50	38	62	72	24	62	68
Lockhart & Martinelli(1949)	124	179	193	27	43	50	200	224	240	28	39	44	29	38	42	31	41	46	27	33	43	42	61	68
Sterman(1956)	27	63	95	10	19	22	103	128	143	10	16	23	12	18	20	10	15	19	1	10	19	27	39	48
Filimonov et al(1957)	147	189	208	26	42	48	125	170	198	30	40	47	26	37	43	31	38	45	25	46	55	64	71	73
Chisholm & Laird(1958)	22	52	80	12	14	16	107	137	157	10	23	30	12	20	23	14	21	29	0	10	22	28	41	51
Flanigan(1958)	12	43	75	3	6	20	79	131	155	4	10	19	0	0	9	0	0	11	9	15	18	10	25	40
Dimentiev et al <sup>2</sup>	34	78	119	16	22	26	93	119	135	9	18	27	5	12	14	9	14	15	9	21	30	30	49	61
Hoogendroon(1959)	0	22	86	2	4	16	90	143	172	4	13	23	0	4	18	0	4	17	12	18	26	9	31	46
Bankoff(1960)	19	50	89	0	2	10	24	53	84	2	5	10	6	11	18	4	10	18	6	8	20	5	7	13
Fauske(1961)	35	56	82	12	14	14	131	148	157	13	21	26	22	32	40	24	36	42	16	28	30	27	39	44
Wilson et al(1961,1965)	34	59	85	6	10	18	21	42	54	6	13	20	8	14	17	0	3	10	3	9	16	9	22	28
Hughmark(1962)	67	180	216	14	41	52	152	208	234	7	36	49	30	47	52	21	47	56	37	51	65	14	66	73
Nicklin et al(1962)	23	79	156	4	20	42	45	87	139	4	14	36	2	20	33	4	18	31	14	40	65	6	17	50
Nishino & Yamazaki(1963)	31	85	129	14	22	32	195	227	236	22	33	41	28	39	44	33	41	49	25	30	37	34	56	60
Fujie(1964)	68	119	159	22	32	54	123	167	194	20	42	53	12	23	31	14	28	35	9	28	56	39	62	63
Kowalczewski(1964)	52	104	144	14	23	26	175	208	218	28	35	39	29	39	47	32	42	48	22	30	37	52	58	63
Thom(1964)	81	126	150	20	27	29	131	186	211	27	34	37	22	32	39	26	37	43	3	21	30	47	55	60
Zivi(1964)	78	117	143	17	18	18	168	190	202	23	32	35	24	31	40	29	37	42	16	26	34	46	55	61
Hughmark(1965)	43	102	171	18	34	50	41	85	142	8	19	44	16	25	37	14	25	40	20	47	71	4	16	44
Neal & Bankoff(1965)	26	51	76	10	12	14	13	24	39	9	10	13	0	3	7	0	5	8	6	9	13	11	19	23
Turner & Wallis(1965)	9	17	40	4	8	12	84	115	134	4	8	14	17	28	33	16	27	36	12	18	26	9	24	32
Baroczy(1966)	52	89	129	16	22	26	183	210	220	21	33	37	28	36	43	30	37	48	22	29	37	36	55	60
Guzhov et al(1967)	0	0	0	0	0	0	0	15	33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Kutucuglu <sup>3</sup>	53	82	104	16	22	26	156	190	210	28	35	40	24	30	37	25	34	40	17	25	30	44	54	57
Smith(1969)	115	171	207	36	46	55	167	209	235	33	43	50	29	36	41	30	41	45	22	46	63	51	58	67
Wallis(1969)	113	171	188	27	49	50	201	227	241	27	38	48	29	37	44	31	40	48	27	41	46	40	62	66
Gregory & Scott(1969)	60	122	197	28	38	54	39	94	185	10	29	57	18	29	45	14	32	50	32	60	73	6	27	63

Table 7. Number of data points correctly predicted for horizontal flow by data sets (contd.)

Correlation / Data Source	Eaton (1966) (Natural Gas - Water)		Beggs (1972) (Air - Water)		Spedding & Nguyen (1976) (Air - Water)		Mukherjee (1979) (Air - Kerosene)		Minami & Brill (1987) (Air - Water)		Minami & Brill (1987) (Air - Kerosene)		Franca & Lahey (1992) (Air - Water)		Abdulmajeed (1996) (Air - Kerosene)									
	Total Points 237		Total Points 56		Total Points 270		Total Points 62		Total Points 54		Total Points 57		Total Points 81		Total Points 83									
	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%						
	162	196	208	27	39	50	199	236	251	26	40	47	33	40	45	40	47	53	38	40	41	62	66	69
Premoli et al(1970)	84	184	221	10	38	50	111	217	240	12	34	48	27	53	54	29	56	57	18	49	61	15	62	72
Rouhani I(1970) <sup>4</sup>	53	164	202	13	26	40	61	101	136	5	20	31	0	4	12	1	6	18	12	28	35	8	27	59
Rouhani II(1970) <sup>4</sup>	25	80	157	4	20	43	45	87	140	4	14	36	2	20	33	5	18	32	14	40	65	6	17	50
Bonnecaze et al(1971)	112	186	210	26	36	44	171	203	220	26	39	47	24	40	46	32	48	51	41	49	66	58	75	78
Dix(1971)	127	188	219	46	50	52	149	185	209	34	43	52	32	42	46	37	45	52	28	49	61	53	66	69
Beggs(1972)	105	173	207	38	54	56	165	206	229	33	45	55	32	38	42	32	42	46	30	56	69	51	64	67
Chisholm(1973)	112	145	154	14	14	19	138	160	173	26	38	45	12	23	29	33	36	45	36	40	41	56	63	66
Loscher & Reinhardt(1973)																								
Mattar & Gregory(1974)	0	1	5	0	0	1	11	29	51	2	3	6	0	0	0	0	0	0	2	11	0	0	0	0
Moussali <sup>5</sup>	68	118	160	17	26	41	109	149	173	22	42	49	12	22	28	14	26	35	17	39	58	34	57	63
Greskovich & Cooper(1975)	101	158	199	30	45	51	128	163	190	31	43	48	24	33	50	28	41	50	24	46	59	43	64	70
Madsen(1975)	2	6	12	0	0	0	24	37	47	0	0	0	0	0	1	0	2	5	0	0	0	0	1	3
Mukherjee(1979)	128	198	222	35	44	46	138	182	218	32	44	47	33	51	54	39	56	57	8	35	55	60	68	74
Gardner-1(1980)	47	81	115	7	18	26	83	131	151	9	18	23	0	1	7	1	7	15	8	14	18	22	37	50
Gardner-2(1980)	82	131	153	13	20	24	97	132	161	17	29	31	2	7	14	19	28	34	12	16	20	44	60	64
Sun et al(1980)	38	107	202	3	16	32	48	82	128	7	20	44	5	23	35	15	34	54	9	29	54	9	28	61
Jowitt(1981)	1	8	15	0	0	3	31	70	175	1	3	7	0	0	0	0	0	0	7	24	36	2	5	10
Chisholm(1983) - Armand(1946)	72	151	203	37	54	56	172	207	229	27	44	55	31	40	44	35	43	49	30	56	69	40	63	67
Spedding & Chen(1984)	33	73	107	14	17	24	204	233	245	23	33	39	27	37	44	31	40	49	28	36	44	34	51	61
Bestion(1985)	89	126	147	14	20	28	58	118	138	12	22	31	0	3	14	0	5	16	14	18	21	22	39	53
Tandon et al(1985)	130	192	215	14	20	26	172	198	205	14	27	36	28	40	47	37	45	54	26	38	41	34	58	66
Chen(1986)	103	139	161	22	42	46	166	206	230	39	46	53	27	33	40	31	37	43	28	48	62	54	60	64
Hammersma & Hart(1987)	37	70	99	12	14	16	107	138	157	10	23	31	12	20	24	14	23	32	0	11	23	29	45	54
Kawaji et al(1987)	50	117	174	16	24	32	133	174	186	17	28	36	26	37	41	29	36	41	16	37	39	27	44	58
Minami & Brill(1987)	131	206	225	33	44	53	149	185	215	33	44	48	36	46	54	48	56	57	27	54	71	61	70	74
El-Boher et al(1988)	16	51	145	15	32	44	162	205	227	10	33	40	14	42	54	1	22	46	45	57	66	18	63	77
Hart et al(1989)	134	184	200	17	20	22	180	192	197	17	26	32	32	43	51	35	52	55	27	38	40	50	63	68
Kokal & Stanislav(1989)	26	80	157	4	20	43	45	88	141	4	14	36	2	20	33	5	18	32	14	40	65	5	17	50
Spedding et al(1989)	34	86	127	14	18	24	185	210	216	21	30	37	28	38	44	31	40	49	27	33	40	37	55	63

Table 7. Number of data points correctly predicted for horizontal flow by data sets (contd.)

Correlation / Data Source	Eaton (1966) (Natural Gas - Water)			Beggs (1972) (Air - Water)			Spedding & Nguyen (1976) (Air - Water)			Mukherjee (1979) (Air - Kerosene)			Minami & Brill (1987) (Air - Water)			Minami & Brill (1987) (Air - Kerosene)			Franca & Lahey (1992) (Air - Water)			Abdulmajeed (1996) (Air - Kerosene)		
	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%
Percentage within	Total Points 237			Total Points 56			Total Points 270			Total Points 62			Total Points 54			Total Points 57			Total Points 81			Total Points 83		
Toshiba(1989)	97	179	209	15	34	44	60	182	221	16	37	45	17	38	47	22	39	48	28	48	59	31	73	75
Huq & Loth(1992)	106	168	206	28	41	51	176	215	239	30	42	47	28	37	41	31	40	45	22	37	60	51	56	65
Inoue et al(1993)	89	165	191	15	26	36	124	164	190	16	31	37	14	27	34	16	30	36	26	38	49	43	56	65
Czop et al(1994)	20	45	107	0	17	24	43	78	126	4	9	30	10	22	38	12	21	41	10	23	40	4	6	28
AbdulMajeed(1996)	70	165	187	26	31	34	104	173	192	23	40	43	19	32	40	22	39	46	2	34	45	48	61	67
Maier and Coddington(1997)	59	136	162	12	21	26	131	166	195	19	31	36	15	29	36	20	34	38	27	41	42	51	56	59
Petalaz & Aziz(1997)	15	30	46	6	6	10	10	17	30	4	8	14	3	6	13	2	3	5	0	2	2	3	7	11
Gomez et al(2000)	52	88	119	2	2	3	96	124	150	0	0	1	14	21	26	0	0	0	0	0	0	8	12	13
Zhao et al(2000)	0	1	3	0	3	6	85	111	121	0	2	4	10	18	22	2	6	10	7	11	19	1	7	11
Graham et al(2001)	137	203	223	20	32	44	181	197	213	22	33	42	36	43	52	40	49	55	29	41	43	55	69	77

<sup>1</sup> correlation as reported by Leung (2005), <sup>2,3</sup> correlations as reported by Kataoka and Ishii (1987), <sup>3</sup> correlation as reported by Isbin and Biddle (1979), <sup>4</sup> Rouhani and Axelsson(1970) correlations I and II.



Table 8. Percentage of data points correctly predicted for horizontal flow by data sets

Correlation / Data Source	Eaton (1966) (Natural Gas - Water)			Beggs (1972) (Air - Water)			Spedding & Nguyen (1976) (Air - Water)			Mukherjee (1979) (Air - Kerosene)			Minami & Brill (1987) (Air - Water)			Minami & Brill (1987) (Air - Kerosene)			Franca & Lahey (1992) (Air - Water)			Abdulmajeed (1996) (Air - Kerosene)		
	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%
Percentage within	Total Points 237			Total Points 56			Total Points 270			Total Points 62			Total Points 54			Total Points 57			Total Points 81			Total Points 83		
Homogeneous	24.5	43.9	61.6	25.0	39.3	62.5	40.0	53.0	61.5	30.6	62.9	75.8	22.2	38.9	51.9	24.6	45.6	57.9	9.9	33.3	60.5	41.0	62.7	74.7
Armand(1946)	22.4	48.9	78.1	41.1	64.3	89.3	15.2	33.3	59.6	16.1	37.1	83.9	27.8	50.0	77.8	24.6	52.6	82.5	35.8	69.1	90.1	6.0	27.7	65.1
Armand - Massina <sup>1</sup>	49.8	78.9	86.5	69.6	89.3	96.4	50.0	81.9	87.4	33.9	71.0	88.7	61.1	77.8	83.3	56.1	77.2	87.7	46.9	76.5	88.9	28.9	74.7	81.9
Lockhart & Martinelli(1949)	52.3	75.5	81.4	48.2	76.8	89.3	74.1	83.0	88.9	45.2	62.9	71.0	53.7	70.4	77.8	54.4	71.9	80.7	33.3	40.7	53.1	50.6	73.5	81.9
Serman(1956)	11.4	26.6	40.1	17.9	33.9	39.3	38.1	47.4	53.0	16.1	25.8	37.1	22.2	33.3	37.0	17.5	26.3	33.3	1.2	12.3	23.5	32.5	47.0	57.8
Filimonov et al(1957)	62.0	79.7	87.8	46.4	75.0	85.7	46.3	63.0	73.3	48.4	64.5	75.8	48.1	68.5	79.6	54.4	66.7	78.9	30.9	56.8	67.9	77.1	85.5	88.0
Chisholm & Laird(1958)	9.3	21.9	33.8	21.4	25.0	28.6	39.6	50.7	58.1	16.1	37.1	48.4	22.2	37.0	42.6	24.6	36.8	50.9	0.0	12.3	27.2	33.7	49.4	61.4
Flanigan(1958)	5.1	18.1	31.6	5.4	10.7	35.7	29.3	48.5	57.4	6.5	16.1	30.6	0.0	0.0	16.7	0.0	0.0	19.3	11.1	18.5	22.2	12.0	30.1	48.2
Dimentiev et al <sup>2</sup>	14.3	32.9	50.2	28.6	39.3	46.4	34.4	44.1	50.0	14.5	29.0	43.5	9.3	22.2	25.9	15.8	24.6	26.3	11.1	25.9	37.0	36.1	59.0	73.5
Hoogendroon(1959)	0.0	9.3	36.3	3.6	7.1	28.6	33.3	53.0	63.7	6.5	21.0	37.1	0.0	7.4	33.3	0.0	7.0	29.8	14.8	22.2	32.1	10.8	37.3	55.4
Bankoff(1960)	8.0	21.1	37.6	0.0	3.6	17.9	8.9	19.6	31.1	3.2	8.1	16.1	11.1	20.4	33.3	7.0	17.5	31.6	7.4	9.9	24.7	6.0	8.4	15.7
Fauske(1961)	14.8	23.6	34.6	21.4	25.0	25.0	48.5	54.8	58.1	21.0	33.9	41.9	40.7	59.3	74.1	42.1	63.2	73.7	19.8	34.6	37.0	32.5	47.0	53.0
Wilson et al(1961,1965)	14.3	24.9	35.9	10.7	17.9	32.1	7.8	15.6	20.0	9.7	21.0	32.3	14.8	25.9	31.5	0.0	5.3	17.5	3.7	11.1	19.8	10.8	26.5	33.7
Hughmark(1962)	28.3	75.9	91.1	25.0	73.2	92.9	56.3	77.0	86.7	11.3	58.1	79.0	55.6	87.0	96.3	36.8	82.5	98.2	45.7	63.0	80.2	16.9	79.5	88.0
Nicklin et al(1962)	9.7	33.3	65.8	7.1	35.7	75.0	16.7	32.2	51.5	6.5	22.6	58.1	3.7	37.0	61.1	7.0	31.6	54.4	17.3	49.4	80.2	7.2	20.5	60.2
Nishino & Yamazaki(1963)	13.1	35.9	54.4	25.0	39.3	57.1	72.2	84.1	87.4	35.5	53.2	66.1	51.9	72.2	81.5	57.9	71.9	86.0	30.9	37.0	45.7	41.0	67.5	72.3
Fujie(1964)	28.7	50.2	67.1	39.3	57.1	96.4	45.6	61.9	71.9	32.3	67.7	85.5	22.2	42.6	57.4	24.6	49.1	61.4	11.1	34.6	69.1	47.0	74.7	75.9
Kowalczewski(1964)	21.9	43.9	60.8	25.0	41.1	46.4	64.8	77.0	80.7	45.2	56.5	62.9	53.7	72.2	87.0	56.1	73.7	84.2	27.2	37.0	45.7	62.7	69.9	75.9
Thom(1964)	34.2	53.2	63.3	35.7	48.2	51.8	48.5	68.9	78.1	43.5	54.8	59.7	40.7	59.3	72.2	45.6	64.9	75.4	3.7	25.9	37.0	56.6	66.3	72.3
Ziv(1964)	32.9	49.4	60.3	30.4	39.3	39.3	62.2	70.4	74.8	37.1	51.6	56.5	44.4	57.4	74.1	50.9	64.9	73.7	19.8	32.1	42.0	55.4	66.3	73.5
Hughmark(1965)	18.1	43.0	72.2	32.1	60.7	89.3	15.2	31.5	52.6	12.9	30.6	71.0	29.6	46.3	68.5	24.6	43.9	70.2	24.7	58.0	87.7	4.8	19.3	53.0
Neal & Bankoff(1965)	11.0	21.5	32.1	17.9	21.4	25.0	4.8	8.9	14.4	14.5	16.1	21.0	0.0	5.6	13.0	0.0	8.8	14.0	7.4	11.1	16.0	13.3	22.9	27.7
Turner & Wallis(1965)	3.8	7.2	16.9	7.1	14.3	21.4	31.1	42.6	49.6	6.5	12.9	22.6	31.5	51.9	61.1	28.1	47.4	63.2	14.8	22.2	32.1	10.8	28.9	38.6
Baroczy(1966)	21.9	37.6	54.4	28.6	39.3	46.4	67.8	77.8	81.5	33.9	53.2	59.7	51.9	66.7	79.6	52.6	64.9	84.2	27.2	35.8	45.7	43.4	66.3	72.3
Guzhov et al(1967)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5.6	12.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Kutucuglu <sup>3</sup>	22.4	34.6	43.9	28.6	39.3	46.4	57.8	70.4	77.8	45.2	56.5	64.5	44.4	55.6	68.5	43.9	59.6	70.2	21.0	30.9	37.0	53.0	65.1	68.7
Smith(1969)	48.5	72.2	87.3	64.3	82.1	98.2	61.9	77.4	87.0	53.2	69.4	80.6	53.7	66.7	75.9	52.6	71.9	78.9	27.2	56.8	77.8	61.4	69.9	80.7
Wallis(1969)	47.7	72.2	79.3	48.2	87.5	89.3	74.4	84.1	89.3	43.5	61.3	77.4	53.7	68.5	81.5	54.4	70.2	84.2	33.3	50.6	56.8	48.2	74.7	79.5
Gregory & Scott(1969)	25.3	51.5	83.1	50.0	67.9	96.4	14.4	34.8	68.5	16.1	46.8	91.9	33.3	53.7	83.3	24.6	56.1	87.7	39.5	74.1	90.1	7.2	32.5	75.9

Table 8. Percentage of data points correctly predicted for horizontal flow by data sets (contd.)

Correlation / Data Source	Eaton (1966) (Natural Gas - Water)			Beggs (1972) (Air - Water)			Spedding & Nguyen (1976) (Air - Water)			Mukherjee (1979) (Air - Kerosene)			Minami & Brill (1987) (Air - Water)			Minami & Brill (1987) (Air - Kerosene)			Franca & Lahey (1992) (Air - Water)			Abdulmajeed (1996) (Air - Kerosene)		
	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%
Percentage within	Total Points 237			Total Points 56			Total Points 270			Total Points 62			Total Points 54			Total Points 57			Total Points 81			Total Points 83		
Premoli et al(1970)	68.4	82.7	87.8	48.2	69.6	89.3	73.7	87.4	93.0	41.9	64.5	75.8	61.1	74.1	83.3	70.2	82.5	93.0	46.9	49.4	50.6	74.7	79.5	83.1
Rouhani I(1970) <sup>4</sup>	35.4	77.6	93.2	17.9	67.9	89.3	41.1	80.4	88.9	19.4	54.8	77.4	50.0	98.1	100.0	50.9	98.2	100.0	22.2	60.5	75.3	18.1	74.7	86.7
Rouhani II(1970) <sup>4</sup>	22.4	69.2	85.2	23.2	46.4	71.4	22.6	37.4	50.4	8.1	32.3	50.0	0.0	7.4	22.2	1.8	10.5	31.6	14.8	34.6	43.2	9.6	32.5	71.1
Bonnecaze et al(1971)	10.5	33.8	66.2	7.1	35.7	76.8	16.7	32.2	51.9	6.5	22.6	58.1	3.7	37.0	61.1	8.8	31.6	56.1	17.3	49.4	80.2	7.2	20.5	60.2
Dix(1971)	47.3	78.5	88.6	46.4	64.3	78.6	63.3	75.2	81.5	41.9	62.9	75.8	44.4	74.1	85.2	56.1	84.2	89.5	50.6	60.5	81.5	69.9	90.4	94.0
Beggs(1972)	53.6	79.3	92.4	82.1	89.3	92.9	55.2	68.5	77.4	54.8	69.4	83.9	59.3	77.8	85.2	64.9	78.9	91.2	34.6	60.5	75.3	63.9	79.5	83.1
Chisholm(1973)	44.3	73.0	87.3	67.9	96.4	100.0	61.1	76.3	84.8	53.2	72.6	88.7	59.3	70.4	77.8	56.1	73.7	80.7	37.0	69.1	85.2	61.4	77.1	80.7
Loscher & Reinhardt(1973)	47.3	61.2	65.0	25.0	25.0	33.9	51.1	59.3	64.1	41.9	61.3	72.6	22.2	42.6	53.7	57.9	63.2	78.9	44.4	49.4	50.6	67.5	75.9	79.5
Mattar & Gregory(1974)	0.0	0.4	2.1	0.0	0.0	1.8	4.1	10.7	18.9	3.2	4.8	9.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.5	13.6	0.0	0.0	0.0
Moussali <sup>5</sup>	28.7	49.8	67.5	30.4	46.4	73.2	40.4	55.2	64.1	35.5	67.7	79.0	22.2	40.7	51.9	24.6	45.6	61.4	21.0	48.1	71.6	41.0	68.7	75.9
Greskovich & Cooper(1975)	42.6	66.7	84.0	53.6	80.4	91.1	47.4	60.4	70.4	50.0	69.4	77.4	44.4	61.1	92.6	49.1	71.9	87.7	29.6	56.8	72.8	51.8	77.1	84.3
Madseen(1975)	0.8	2.5	5.1	0.0	0.0	0.0	8.9	13.7	17.4	0.0	0.0	0.0	0.0	0.0	1.9	0.0	3.5	8.8	0.0	0.0	0.0	0.0	1.2	3.6
Mukherjee(1979)	54.0	83.5	93.7	62.5	78.6	82.1	51.1	67.4	80.7	51.6	71.0	75.8	61.1	94.4	100.0	68.4	98.2	100.0	9.9	43.2	67.9	72.3	81.9	89.2
Gardner-1(1980)	19.8	34.2	48.5	12.5	32.1	46.4	30.7	48.5	55.9	14.5	29.0	37.1	0.0	1.9	13.0	1.8	12.3	26.3	9.9	17.3	22.2	26.5	44.6	60.2
Gardner-2(1980)	34.6	55.3	64.6	23.2	35.7	42.9	35.9	48.9	59.6	27.4	46.8	50.0	3.7	13.0	25.9	33.3	49.1	59.6	14.8	19.8	24.7	53.0	72.3	77.1
Sun et al(1980)	16.0	45.1	85.2	5.4	28.6	57.1	17.8	30.4	47.4	11.3	32.3	71.0	9.3	42.6	64.8	26.3	59.6	94.7	11.1	35.8	66.7	10.8	33.7	73.5
Jowitt(1981)	0.4	3.4	6.3	0.0	0.0	5.4	11.5	25.9	64.8	1.6	4.8	11.3	0.0	0.0	0.0	0.0	0.0	0.0	8.6	29.6	44.4	2.4	6.0	12.0
Chisholm(1983) - Armand(1946)	30.4	63.7	85.7	66.1	96.4	100.0	63.7	76.7	84.8	43.5	71.0	88.7	57.4	74.1	81.5	61.4	75.4	86.0	37.0	69.1	85.2	48.2	75.9	80.7
Spedding & Chen(1984)	13.9	30.8	45.1	25.0	30.4	42.9	75.6	86.3	90.7	37.1	53.2	62.9	50.0	68.5	81.5	54.4	70.2	86.0	34.6	44.4	54.3	41.0	61.4	73.5
Bestion(1985)	37.6	53.2	62.0	25.0	35.7	50.0	21.5	43.7	51.1	19.4	35.5	50.0	0.0	5.6	25.9	0.0	8.8	28.1	17.3	22.2	25.9	26.5	47.0	63.9
Tandon et al(1985)	54.9	81.0	90.7	25.0	35.7	46.4	63.7	73.3	75.9	22.6	43.5	58.1	51.9	74.1	87.0	64.9	78.9	94.7	32.1	46.9	50.6	41.0	69.9	79.5
Chen(1986)	43.5	58.6	67.9	39.3	75.0	82.1	61.5	76.3	85.2	62.9	74.2	85.5	50.0	61.1	74.1	54.4	64.9	75.4	34.6	59.3	76.5	65.1	72.3	77.1
Hamersma & Hart(1987)	15.6	29.5	41.8	21.4	25.0	28.6	39.6	51.1	58.1	16.1	37.1	50.0	22.2	37.0	44.4	24.6	40.4	56.1	0.0	13.6	28.4	34.9	54.2	65.1
Kawaji et al(1987)	21.1	49.4	73.4	28.6	42.9	57.1	49.3	64.4	68.9	27.4	45.2	58.1	48.1	68.5	75.9	50.9	63.2	71.9	19.8	45.7	48.1	32.5	53.0	69.9
Minami & Brill(1987)	55.3	86.9	94.9	58.9	78.6	94.6	55.2	68.5	79.6	53.2	71.0	77.4	66.7	85.2	100.0	84.2	98.2	100.0	33.3	66.7	87.7	73.5	84.3	89.2
El-Boher et al(1988)	6.8	21.5	61.2	26.8	57.1	78.6	60.0	75.9	84.1	16.1	53.2	64.5	25.9	77.8	100.0	1.8	38.6	80.7	55.6	70.4	81.5	21.7	75.9	92.8
Hart et al(1989)	56.5	77.6	84.4	30.4	35.7	39.3	66.7	71.1	73.0	27.4	41.9	51.6	59.3	79.6	94.4	61.4	91.2	96.5	33.3	46.9	49.4	60.2	75.9	81.9
Kokal & Stanislav(1989)	11.0	33.8	66.2	7.1	35.7	76.8	16.7	32.6	52.2	6.5	22.6	58.1	3.7	37.0	61.1	8.8	31.6	56.1	17.3	49.4	80.2	6.0	20.5	60.2
Spedding et al(1989)	14.3	36.3	53.6	25.0	32.1	42.9	68.5	77.8	80.0	33.9	48.4	59.7	51.9	70.4	81.5	54.4	70.2	86.0	33.3	40.7	49.4	44.6	66.3	75.9

Table 8. Percentage of data points correctly predicted for horizontal flow by data sets (contd.)

Correlation / Data Source	Eaton (1966) (Natural Gas - Water)		Beggs (1972) (Air -Water)		Spedding & Nguyen (1976) (Air -Water)		Mukherjee (1979) (Air -Kerosene)		Minami & Brill (1987) (Air -Water)		Minami & Brill (1987) (Air -Kerosene)		Franca & Lahey (1992) (Air -Water)		Abdulmajeed (1996) (Air -Kerosene)									
	Total Points 237		Total Points 56		Total Points 270		Total Points 62		Total Points 54		Total Points 57		Total Points 81		Total Points 83									
	±5%	±10%	±5%	±10%	±5%	±10%	±5%	±10%	±5%	±10%	±5%	±10%	±5%	±10%	±5%	±10%								
Toshiba(1989)	40.9	75.5	88.2	26.8	60.7	78.6	22.2	67.4	81.9	72.6	31.5	70.4	87.0	34.6	59.3	72.8	37.3	88.0	90.4					
Huq & Loth(1992)	44.7	70.9	86.9	50.0	73.2	91.1	65.2	79.6	88.5	48.4	67.7	75.8	51.9	68.5	75.9	54.4	70.2	78.9	27.2	45.7	74.1	61.4	67.5	78.3
Inoue et al(1993)	37.6	69.6	80.6	26.8	46.4	64.3	45.9	60.7	70.4	25.8	50.0	59.7	25.9	50.0	63.0	28.1	52.6	63.2	32.1	46.9	60.5	51.8	67.5	78.3
Czop et al(1994)	8.4	19.0	45.1	0.0	30.4	42.9	15.9	28.9	46.7	6.5	14.5	48.4	18.5	40.7	70.4	21.1	36.8	71.9	12.3	28.4	49.4	4.8	7.2	33.7
AbdulMajeed(1996)	29.5	69.6	78.9	46.4	55.4	60.7	38.5	64.1	71.1	37.1	64.5	69.4	35.2	59.3	74.1	38.6	68.4	80.7	2.5	42.0	55.6	57.8	73.5	80.7
Maier and Coddington(1997)	24.9	57.4	68.4	21.4	37.5	46.4	48.5	61.5	72.2	30.6	50.0	58.1	27.8	53.7	66.7	35.1	59.6	66.7	33.3	50.6	51.9	61.4	67.5	71.1
Petalaz & Aziz(1997)	6.3	12.7	19.4	10.7	10.7	17.9	3.7	6.3	11.1	6.5	12.9	22.6	5.6	11.1	24.1	3.5	5.3	8.8	0.0	2.5	2.5	3.6	8.4	13.3
Gomez et al(2000)	21.9	37.1	50.2	3.6	3.6	5.4	35.6	45.9	55.6	0.0	0.0	1.6	25.9	38.9	48.1	0.0	0.0	0.0	0.0	0.0	0.0	9.6	14.5	15.7
Zhao et al(2000)	0.0	0.4	1.3	0.0	5.4	10.7	31.5	41.1	44.8	0.0	3.2	6.5	18.5	33.3	40.7	3.5	10.5	17.5	8.6	13.6	23.5	1.2	8.4	13.3
Graham et al(2001)	57.8	85.7	94.1	35.7	57.1	78.6	67.0	73.0	78.9	35.5	53.2	67.7	66.7	79.6	96.3	70.2	86.0	96.5	35.8	50.6	53.1	66.3	83.1	92.8

correlation as reported by Leung (2005), <sup>2,3</sup> correlations as reported by Kataoka and Ishii (1987), <sup>3</sup> correlation as reported by Isbin and Biddle (1979), <sup>4</sup> Rouhani and Axelsson(1970) correlations I and II.

A simple scatter plot of the measured versus calculated void fractions for the top six correlations that predicted 85% of the data points in the horizontal flow data base within the 15% error percentage index is presented. These are Rouhani I(1970), Hughmark (1962), Minami and Brill (1987), Armand–Massina (Leung 2005), Mukherjee(1979), Chisholm(1973) and Chisholm(1983) – Armand (1946) that are given in Figures 1 through 6, respectively. The purpose of these plots is to show where the rest of the “stray” data points that could not be predicted by the correlations lie. This would reveal how much consistent the prediction by the correlation is and also the general tendency to under or over predict a given data set through the range of void fraction values.

We can see that the correlation of Hughmark (1962), which is the second best correlation in terms of the total percentage of data points predicted, shows a superior consistent performance over the others. Rouhani I( 1970) correlation is seen to under predict the data most of the time while all the other correlations tend to over predict the data on the whole range of values.

A final note on this analysis is that one could opt for a correlation with a fairly consistent performance across the whole void fraction range even when the correlation may not give accurate prediction of most of the data. On the other hand, a specific correlation which shows good performance in a specific range could be a viable choice if the range of voidage encountered for the particular application of interest has been indirectly determined or expected to be in that area.

Considering predicting void fraction for air –water mixture for example, sixteen correlations are observed to give a prediction of 85% and above for the 15% error index with the best predictions coming from Armand Massina (Leung 2005), Rouhani I(1970) and Hughmark(1962) with predictions of 87% and above. A very close

performance to these correlations were also given by Chisholm(1973) and Chisholm(1983)–Armand(1946) which predicted 86% while Smith(1969),Minami & Brill(1987), El-Boher et al.(1988) and Huq & Loth(1992) captured 85% of the data within the 15% error index.

On another hand, for the non air-water data, the correlations of Beggs (1972) and Graham et al. (2001) gave the best prediction of 89 and 88%, respectively. This is quite unexpected as these correlations were developed from air water and refrigerant data sets that have quite different physical properties from kerosene and natural gas. Rouhani I (1970) and Toshiba (1989) predicted 86% of the data set for the 15% index with close predictions of 85% coming from Armand Massina (Leung 2005), Filimonov et al. (1957), Chisholm (1973), Premoli et al. (1970) and Chisholm (1983)-Armand (1946).

We would now move on to the comparison of the correlations with the inclined flow database compiled.

#### **4.2 Inclined flow comparison**

In this section we would use data sets from three different sources that reported void fraction data for different upward inclination angles. As the inclination angles investigated by the different researchers are not the same, the results from one data set would not be directly comparable to the other except on some inclination angles which are common to all. Hence, the presentation of the comparison results here would be a bit different from the previous section in the sense that it would be restricted to a single data source at a time.

We would therefore subdivide this section into the three different data sources we have, namely, Beggs (1972), Spedding and Nguyen (1976) and Mukherjee (1979)

inclined flow comparisons. All the inclination angles within the data set would then be considered separately first before making a consolidated summary for all.

A few of the correlations have inclination angle dependent terms or constant in their expression which has to be adjusted for the individual inclinations angles under consideration to predict the void fraction. These are the correlation by Beggs (1972), Greskovich and Cooper (1975) and Gomez et al. (2000).

#### **4.2.1 Inclined flow comparison with the data of Beggs(1972)**

Beggs (1972) reported 209 void fraction measurements for seven different pipe inclination angles between 5 – 75 degrees from the horizontal. The mixtures used were air and water in 1 and 1.5 in diameter pipes. The total number of data points correctly predicted for each pipe inclination angle is given in Table 10 while the percentage comparisons are reported in Table 11.

Owing to the fact that there are only a small number of data points and the inclination is very close to the horizontal, four correlations were seen to perform acceptably with the tightest 5% error index for the 5 degree inclination data set.

These are the simpler models by Armand Massina, Chisholm (1973), Chisholm (1983) – Armand (1946) and the correlation by Beggs (1972). As was observed in the horizontal comparison, the performance of the correlations improve for the wider error bands of 10 and 15 % where 12 and 21 correlations were found to predict at least 80 and 85% of the data sets respectively. The same trend is observed for all inclination angles except for a few exceptions.

Table 9. Total number and percentage of data points correctly predicted for horizontal flow database (all sources)

Correlation / Data Source	Total data points correctly predicted for the whole database			Percentage of data points correctly predicted for the whole database		
	Total Points 900			Total Points 900		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%
Homogeneous	267	434	566	29.7	48.2	62.9
Armand(1946)	190	401	664	21.1	44.6	73.8
Armand - Massina <sup>1</sup>	440	712	785	48.9	79.1	87.2
Lockhart & Martinelli(1949)	508	658	726	56.4	73.1	80.7
Sterman(1956)	200	308	389	22.2	34.2	43.2
Filimonov et al(1957)	474	633	717	52.7	70.3	79.7
Chisholm & Laird(1958)	205	318	408	22.8	35.3	45.3
Flanigan(1958)	117	230	347	13.0	25.6	38.6
Dimentiev et al <sup>2</sup>	205	333	427	22.8	37.0	47.4
Hoogendroon(1959)	117	239	404	13.0	26.6	44.9
Bankoff(1960)	66	146	262	7.3	16.2	29.1
Fauske(1961)	280	374	435	31.1	41.6	48.3
Wilson et al(1961,1965)	87	172	248	9.7	19.1	27.6
Hughmark(1962)	342	676	797	38.0	75.1	88.6
Nicklin et al(1962)	102	295	552	11.3	32.8	61.3
Nishino & Yamazaki(1963)	382	533	628	42.4	59.2	69.8
Fujie(1964)	307	501	645	34.1	55.7	71.7
Kowalczewski(1964)	404	539	622	44.9	59.9	69.1
Thom(1964)	357	518	599	39.7	57.6	66.6
Zivi(1964)	401	510	579	44.6	56.7	64.3
Hughmark(1965)	164	353	599	18.2	39.2	66.6
Neal & Bankoff(1965)	75	133	193	8.3	14.8	21.4
Turner & Wallis(1965)	155	245	327	17.2	27.2	36.3
Baroczy(1966)	388	511	600	43.1	56.8	66.7
Guzhov et al(1967)	0	15	33	0.0	1.7	3.7
Kutucuglu <sup>3</sup>	363	472	544	40.3	52.4	60.4
Smith(1969)	483	650	763	53.7	72.2	84.8
Wallis(1969)	495	665	731	55.0	73.9	81.2
Gregory & Scott(1969)	207	431	724	23.0	47.9	80.4
Premoli et al(1970)	587	704	764	65.2	78.2	84.9
Rouhani I(1970) <sup>4</sup>	306	693	803	34.0	77.0	89.2
Rouhani II(1970) <sup>4</sup>	153	376	533	17.0	41.8	59.2
Bonnecaze et al(1971)	105	296	556	11.7	32.9	61.8
Dix(1971)	490	676	762	54.4	75.1	84.7
Beggs(1972)	506	668	760	56.2	74.2	84.4
Chisholm(1973)	486	678	771	54.0	75.3	85.7
Loscher & Reinhardt(1973)	427	519	572	47.4	57.7	63.6
Mattar & Gregory(1974)	13	35	74	1.4	3.9	8.2
Moussali <sup>5</sup>	293	479	607	32.6	53.2	67.4
Greskovich & Cooper(1975)	409	593	717	45.4	65.9	79.7
Madsen(1975)	26	46	68	2.9	5.1	7.6
Mukherjee(1979)	473	678	773	52.6	75.3	85.9



Table 9. Total number and percentage of data points correctly predicted for horizontal flow database (all sources) (contd.)

Correlation / Data Source	Total data points correctly predicted for the whole database			Percentage of data points correctly predicted for the whole database		
	Total Points 900			Total Points 900		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%
Gardner-1(1980)	177	307	405	19.7	34.1	45.0
Gardner-2(1980)	286	423	501	31.8	47.0	55.7
Sun et al(1980)	134	339	610	14.9	37.7	67.8
Jowitt(1981)	42	110	246	4.7	12.2	27.3
Chisholm(1983) - Armand(1946)	444	658	772	49.3	73.1	85.8
Spedding & Chen(1984)	394	520	613	43.8	57.8	68.1
Bestion(1985)	209	351	448	23.2	39.0	49.8
Tandon et al(1985)	455	618	690	50.6	68.7	76.7
Chen(1986)	470	611	699	52.2	67.9	77.7
Hamersma & Hart(1987)	221	344	436	24.6	38.2	48.4
Kawaji et al(1987)	314	497	607	34.9	55.2	67.4
Minami & Brill(1987)	518	705	797	57.6	78.3	88.6
El-Boher et al(1988)	281	505	699	31.2	56.1	77.7
Hart et al(1989)	492	618	665	54.7	68.7	73.9
Kokal & Stanislav(1989)	105	297	557	11.7	33.0	61.9
Spedding et al(1989)	377	510	600	41.9	56.7	66.7
Toshiba(1989)	286	630	748	31.8	70.0	83.1
Huq & Loth(1992)	472	636	754	52.4	70.7	83.8
Inoue et al(1993)	343	537	638	38.1	59.7	70.9
Czop et al(1994)	103	221	434	11.4	24.6	48.2
AbdulMajeed(1996)	314	575	654	34.9	63.9	72.7
Maier and Coddington(1997)	334	514	594	37.1	57.1	66.0
Petalaz & Aziz(1997)	43	79	131	4.8	8.8	14.6
Gomez et al(2000)	172	247	312	19.1	27.4	34.7
Zhao et al(2000)	105	159	196	11.7	17.7	21.8
Graham et al(2001)	520	667	749	57.8	74.1	83.2

<sup>1</sup> correlation as reported by Leung (2005), <sup>2,5</sup> correlations as reported by Kataoka and Ishii (1987), <sup>3</sup> correlation as reported by Isbin and Biddle (1979), <sup>4</sup> Rouhani and Axelsson(1970) correlations I and II.

The Minami and Brill (1987) correlation fairly predicted the 10 and 20 degree data within the 5% index while Chisholm (1973), Chisholm (1983) Armand (1946) and Smith (1969) also gave similar results for the 15 degree data. A consistent acceptable prediction for the high inclination angle ranges (35 -75) comes from Dix (1971) correlation which predicted the data for all error indices quite well.



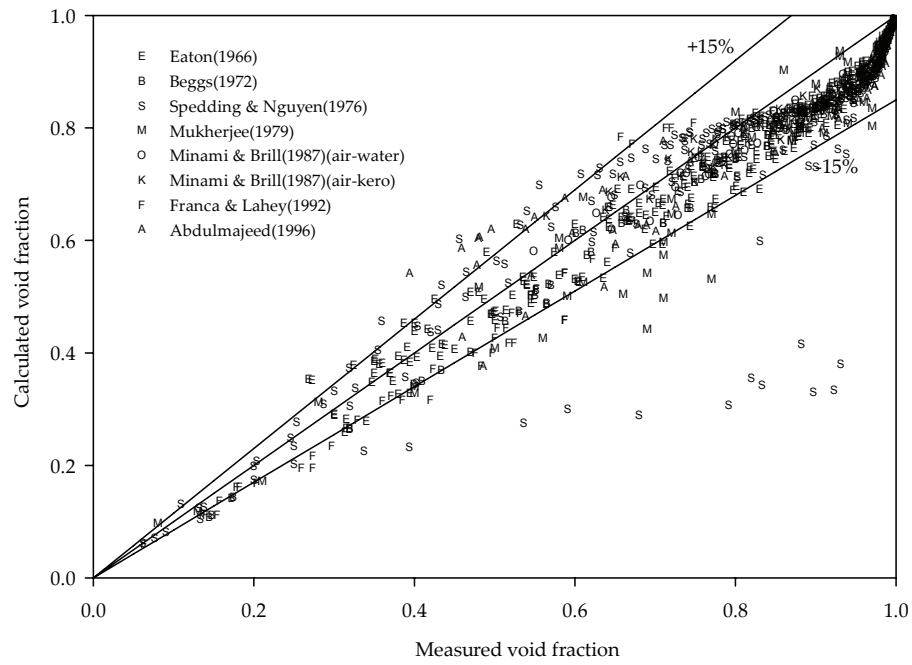


Figure 1. Comparison of Rouhani I (1970) correlation with measured horizontal experimental data

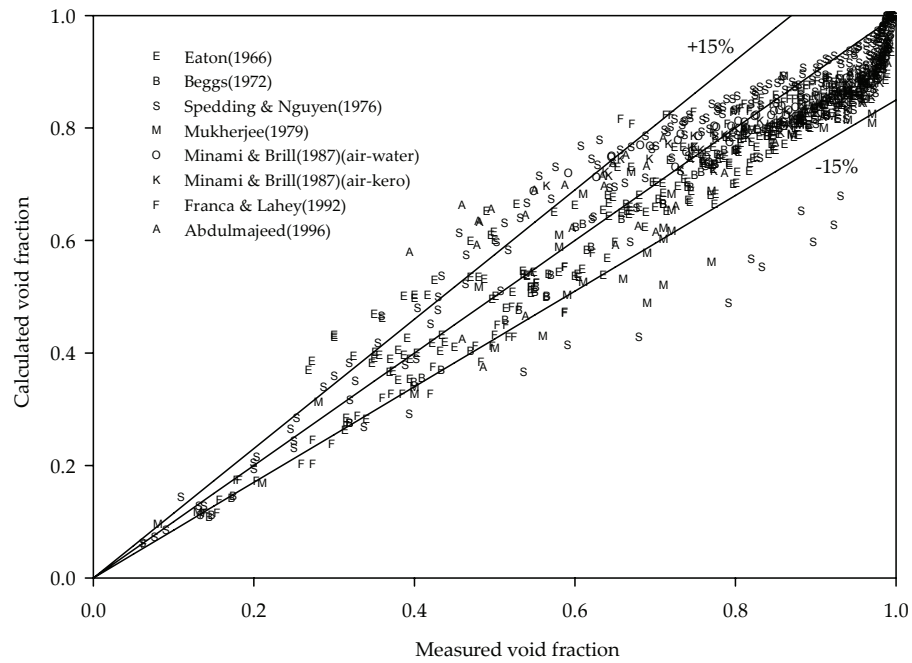


Figure 2. Comparison of Hughmark (1962) correlation with measured horizontal experimental data

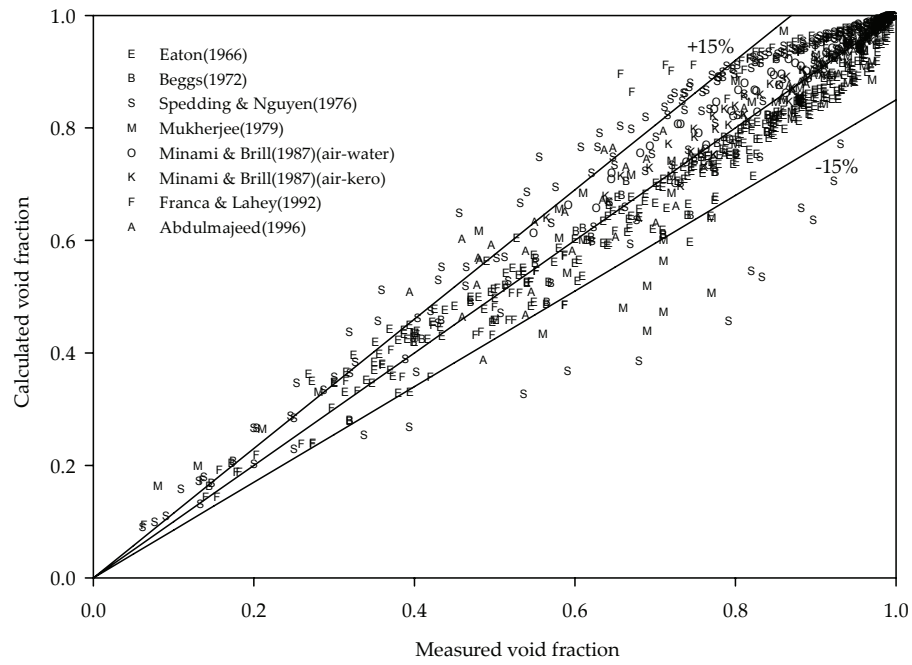


Figure 3. Comparison of Minami and Brill (1987) correlation with measured horizontal experimental data

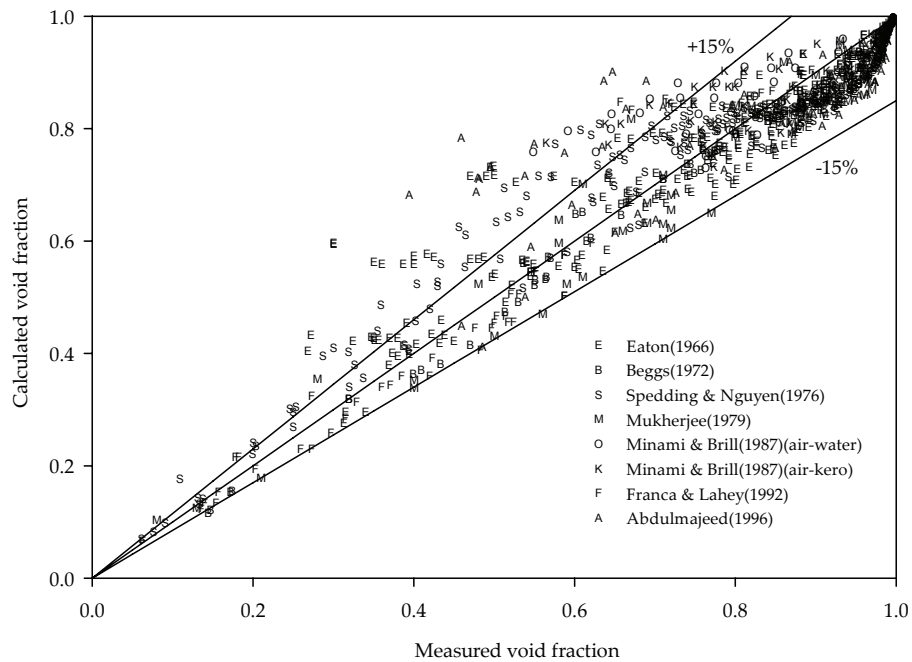


Figure 4. Comparison of Armand Massina correlation (Leung 2005) with measured horizontal experimental data

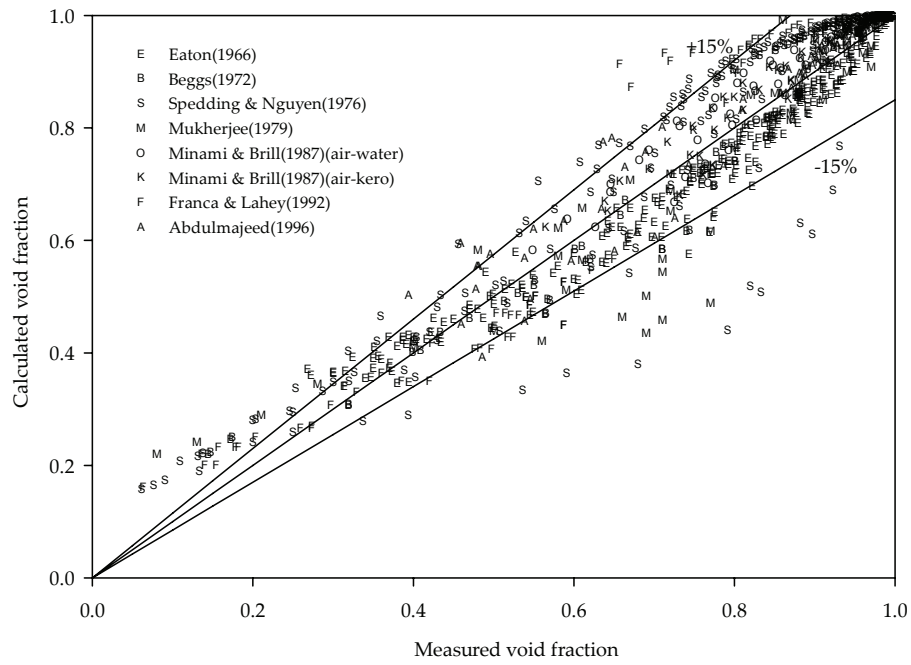


Figure 5. Comparison of Mukherjee (1979) correlation with measured horizontal experimental data

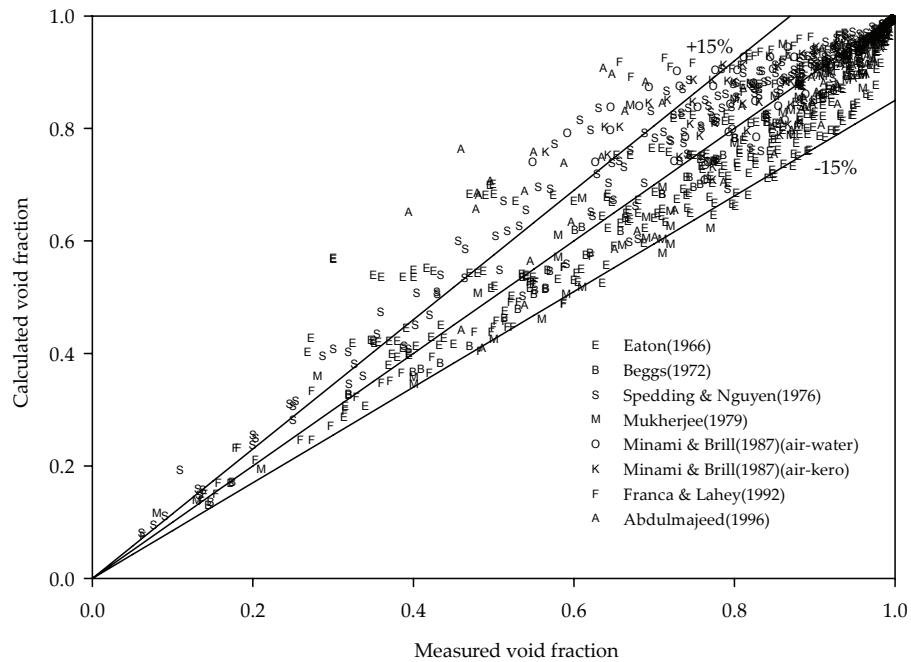


Figure 6. Comparison of Chisholm (1983) – Armand (1946) correlation with measured horizontal experimental data

A combined assessment for all the inclination angles was performed and the results of which are given in Table 12. Excellent prediction of the data was given by Chisholm (1983)–Armand (1946), Chisholm (1973), Gregory & Scott (1969), Hughmark (1962) and Armand Massina (Leung 2005). The correlations by Filimonov et al. (1957), Premoli et al. (1970), Rouhani I (1970), Dix (1971) and Toshiba (1989) gave comparable results to the other set of correlations mentioned above. Among these correlations the Hughmark (1962) correlation gave the best prediction of 96.7% of the data within the 15% index while Premoli et al. (1970), Rouhani I (1970), Filimonov et al. (1957), Toshiba (1989) and Dix (1971) gave 96.2, 95.7, 95.2, 94.3 and 91.4 percent respectively. A scatter plot of the performance of each of the correlations in the latter group is shown for each inclination angle in Figures 7 to 13. The decision to consider these correlations is driven primarily due to the fact that these correlations would be shown to predict inclined data from other sources very well. On the other hand those correlations that did perform superior for the Beggs (1972) inclined data did not do as much on the other data sets.

#### **4.2.2 Inclined flow comparison with the data of Spedding and Nguyen (1976)**

An extensive void fraction data restricted to only four inclination angles was given by Spedding and Nguyen (1976). The angles considered were 2.75, 20.75, 45 and 75 degrees. A total of 889 void fraction measurements were reported for which the number and percentage of data points of predictions by the correlations are reported in Table 13 and Table 14 respectively.

This data set is where almost all the predictions by the correlations fall short of being acceptable in all the error indices across all inclination angles.

Table 10. Number of data points correctly predicted for Beggs (1972) Inclined data

Correlation / Inclination Angle	5 Degree data			10 Degree data			15 Degree data			20 Degree data			35 Degree data			55 Degree data			75 Degree data		
	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%
Percentage within	Total Points 31			Total Points 29			Total Points 31			Total Points 30			Total Points 30			Total Points 29			Total Points 29		
Homogeneous	4	10	14	4	11	13	4	8	11	4	6	10	4	7	10	6	8	12	5	9	12
Armand(1946)	18	23	29	13	21	27	18	24	29	14	22	28	14	19	27	12	16	24	12	19	26
Armand - Massina <sup>1</sup>	23	31	31	16	28	29	20	30	31	16	26	30	17	21	28	14	20	25	16	23	28
Lockhart & Martinelli(1949)	18	27	28	16	25	26	17	25	29	15	24	29	12	22	27	11	22	25	13	22	27
Sterman(1956)	7	11	12	6	10	11	5	11	12	6	11	11	5	10	12	6	12	13	8	11	12
Filimonov et al(1957)	17	26	30	18	26	28	15	24	30	20	26	28	15	25	28	16	25	27	17	25	28
Chisholm & Laird(1958)	4	7	8	4	7	8	4	6	7	3	5	6	3	4	6	3	4	7	4	5	7
Flanigan(1958)	2	4	9	3	6	10	2	6	11	1	4	9	0	4	10	2	8	12	3	5	10
Dimentiev et al <sup>2</sup>	7	12	16	8	11	15	8	11	15	7	13	15	8	12	14	8	11	15	8	10	15
Hoogendoorn(1959)	0	4	9	0	5	11	1	5	11	0	4	11	0	3	13	0	4	12	0	4	10
Bankoff(1960)	0	1	10	0	4	10	0	4	16	4	6	17	4	7	17	5	7	15	3	5	12
Fauske(1961)	6	7	9	6	7	8	6	7	8	4	4	9	11	4	9	3	8	9	5	8	9
Wilson et al(1961,1965)	4	8	9	1	6	7	2	7	8	3	9	9	3	10	12	1	8	12	2	8	11
Hughmark(1962)	14	28	30	14	25	28	18	28	31	15	27	30	13	26	29	12	25	27	12	25	27
Nicklin et al(1962)	2	18	27	7	17	26	10	21	29	15	21	28	15	20	27	13	20	24	12	18	25
Nishino & Yamazaki(1963)	10	15	18	10	14	16	9	16	18	12	17	21	10	16	20	11	16	19	12	14	19
Fujie(1964)	10	16	26	9	16	21	8	14	20	7	13	16	6	12	16	7	10	17	9	14	17
Kowalczewski(1964)	12	14	17	9	13	15	9	15	16	12	15	18	8	15	17	8	15	17	11	15	17
Thom(1964)	11	16	19	10	13	17	9	14	15	8	14	17	8	13	15	9	12	16	10	13	16
Zivi(1964)	11	13	15	7	11	15	7	12	15	7	13	16	7	9	15	6	12	15	7	12	15
Hughmark(1965)	13	23	29	14	18	27	19	23	29	15	22	28	14	19	27	11	17	25	12	18	25
Neal & Bankoff(1965)	4	7	8	3	7	8	3	7	9	3	5	8	3	6	7	3	6	9	2	4	6
Turner & Wallis(1965)	2	4	6	3	4	6	3	5	6	4	5	6	4	5	6	4	5	6	4	5	7
Baroczy(1966)	10	13	17	9	12	15	9	14	16	9	16	18	8	13	17	8	14	16	11	14	16
Guzhov et al(1967)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Kutucuglu <sup>3</sup>	9	15	15	9	10	14	10	13	15	7	12	16	8	12	13	8	12	13	8	12	13
Smith(1969)	18	29	31	17	26	27	23	28	31	14	27	29	12	23	26	15	20	25	10	23	26
Wallis(1969)	18	25	27	16	24	25	21	25	26	14	23	26	13	20	26	12	20	25	14	22	27

Table 10. Number of data points correctly predicted for Beggs (1972) Inclined data (contd.)

Correlation / Inclination Angle	5 Degree data			10 Degree data			15 Degree data			20 Degree data			35 Degree data			55 Degree data			75 Degree data		
	Total Points 31			Total Points 29			Total Points 31			Total Points 30			Total Points 30			Total Points 29			Total Points 29		
	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%
Percentage within	17	23	31	12	22	29	16	25	31	15	22	30	13	18	27	11	18	25	12	20	29
Gregory & Scott(1969)	19	26	30	17	26	27	19	29	31	20	27	30	17	25	28	17	22	27	18	22	28
Premoli et al(1970)	9	26	29	14	25	28	18	27	31	20	26	30	20	26	29	17	26	26	18	25	27
Rouhani I(1970) <sup>4</sup>	9	18	23	9	18	25	8	23	27	10	19	28	12	21	27	8	21	28	10	19	27
Rouhani II(1970) <sup>4</sup>	2	18	27	7	17	26	10	21	29	15	21	28	15	20	27	13	20	24	12	18	25
Bonnecaze et al(1971)	14	24	26	16	24	26	20	26	27	20	25	28	23	28	29	22	28	28	22	26	27
Dix(1971)	26	27	28	19	26	27	21	24	27	14	20	27	13	20	27	12	19	25	16	21	28
Beggs(1972)	25	29	31	20	27	29	23	29	31	17	28	30	12	21	30	15	20	27	14	25	29
Chisholm(1973)	7	8	13	7	9	11	7	9	10	6	9	15	8	9	13	7	9	13	7	9	13
Loscher & Reinhardt(1973)	0	0	1	0	0	0	0	0	2	0	0	2	0	0	2	0	0	2	0	0	1
Matiar & Gregory(1974)	9	14	21	8	14	17	7	12	15	7	9	13	5	9	13	5	10	13	5	12	15
Moussali <sup>5</sup>	19	25	30	14	21	28	14	20	30	9	16	28	7	13	27	6	16	25	8	16	25
Greskovich & Cooper(1975)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Madsen(1975)	22	24	28	19	25	28	19	26	28	18	28	28	21	28	29	20	26	28	17	27	28
Mukherjee(1979)	5	9	15	6	11	15	5	8	14	3	6	13	3	9	14	6	11	16	6	9	15
Gardner-1(1980)	7	12	14	7	11	13	4	10	13	6	11	15	4	11	16	5	11	16	3	13	16
Gardner-2(1980)	2	12	23	2	16	22	7	18	24	10	20	26	13	20	24	12	18	23	9	16	22
Sun et al(1980)	0	0	0	0	0	0	0	0	0	0	0	1	0	0	4	0	0	4	0	0	4
Jowitt(1981)	24	29	31	18	27	29	23	31	31	19	28	30	15	22	30	16	20	29	16	25	28
Chisholm(1983) - Armand(1946)	10	13	18	9	11	17	7	13	18	9	16	18	8	14	19	6	14	17	8	15	16
Spedding & Chen(1984)	8	13	17	9	12	18	9	14	18	10	13	19	11	15	19	10	15	20	9	13	18
Bestion(1985)	9	13	18	9	13	16	9	14	18	12	16	19	10	15	19	7	15	19	9	15	18
Tandon et al(1985)	11	19	25	12	16	22	7	13	22	6	13	18	8	13	18	7	14	18	6	13	20
Chen(1986)	4	8	8	4	7	8	4	7	7	3	5	6	3	4	6	4	5	7	4	5	7
Hamersma & Hart(1987)	10	14	16	9	12	16	7	12	17	9	16	20	8	15	20	6	17	20	8	14	17
Kawaji et al(1987)	20	26	28	21	25	27	20	25	28	21	27	28	20	28	29	19	26	28	19	26	28
Minami & Brill(1987)	4	21	25	8	21	25	12	21	25	14	26	27	16	28	28	13	25	26	15	22	25
El-Boher et al(1988)	10	12	14	9	12	15	9	12	14	11	14	17	8	12	15	8	11	16	7	12	16
Hart et al(1989)																					

Table 10. Number of data points correctly predicted for Beggs (1972) Inclined data (contd.)

Correlation / Inclination Angle	5 Degree data			10 Degree data			15 Degree data			20 Degree data			35 Degree data			55 Degree data			75 Degree data		
	Total Points 31			Total Points 29			Total Points 31			Total Points 30			Total Points 30			Total Points 29			Total Points 29		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%
Kokal & Stanislav(1989)	2	18	27	7	17	26	10	21	29	15	21	28	15	20	27	13	20	24	12	18	25
Spedding et al(1989)	9	12	16	9	12	16	7	12	16	9	16	17	8	14	16	6	14	16	8	15	16
Toshiba(1989)	11	26	29	13	26	28	16	25	29	17	28	29	21	27	28	18	27	27	16	25	27
Huq & Loth(1992)	17	26	30	12	25	27	18	27	31	14	26	29	12	23	26	11	23	24	10	21	25
Inoue et al(1993)	10	17	23	11	18	24	11	20	24	12	23	26	13	23	26	13	23	26	11	22	25
Czop et al(1994)	2	9	16	4	7	17	5	10	17	9	13	19	6	15	20	6	14	18	7	12	16
AbdulMajeed(1996)	11	18	20	9	17	19	8	16	19	8	17	21	10	16	20	11	15	19	11	18	20
Maier and Coddington(1997)	7	12	19	7	13	16	7	13	16	9	17	19	12	15	18	10	16	17	10	16	16
Petalaz & Aziz(1997)	2	3	6	2	6	7	3	5	6	1	4	5	2	5	8	1	6	7	2	6	7
Gomez et al(2000)	2	2	2	0	1	1	0	1	2	0	0	0	0	1	1	0	1	1	1	1	1
Zhao et al(2000)	0	2	3	0	2	3	0	2	3	0	2	3	0	2	4	0	2	3	0	2	3
Graham et al(2001)	14	21	26	13	19	27	13	25	29	14	27	28	15	25	28	14	25	26	12	24	27

<sup>1</sup> correlation as reported by Leung (2005), <sup>2,5</sup> correlations as reported by Kataoka and Ishii (1987), <sup>3</sup> correlation as reported by Isbin and Biddle (1979), <sup>4</sup> Rouhani and Axelsson(1970) correlations I and II.

Table 11. Percentage of data points correctly predicted for Beggs (1972) Inclined data

Correlation / Inclination Angle	5 Degree data			10 Degree data			15 Degree data			20 Degree data			35 Degree data			55 Degree data			75 Degree data		
	Total Points 31			Total Points 29			Total Points 31			Total Points 30			Total Points 30			Total Points 29			Total Points 29		
	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%
Percentage within																					
Homogeneous	12.9	32.3	45.2	13.8	37.9	44.8	12.9	25.8	35.5	13.3	20.0	33.3	13.3	23.3	33.3	20.7	27.6	41.4	17.2	31.0	41.4
Armand(1946)	58.1	74.2	93.5	44.8	72.4	93.1	58.1	77.4	93.5	46.7	73.3	93.3	46.7	63.3	90.0	41.4	55.2	82.8	41.4	65.5	89.7
Armand - Massina <sup>1</sup>	74.2	100.0	100.0	55.2	96.6	100.0	64.5	96.8	100.0	53.3	86.7	100.0	56.7	70.0	93.3	48.3	69.0	86.2	55.2	79.3	96.6
Lockhart & Martinelli(1949)	58.1	87.1	90.3	55.2	86.2	89.7	54.8	80.6	93.5	50.0	80.0	96.7	40.0	73.3	90.0	37.9	75.9	86.2	44.8	75.9	93.1
Sterman(1956)	22.6	35.5	38.7	20.7	34.5	37.9	16.1	35.5	38.7	20.0	36.7	36.7	16.7	33.3	40.0	20.7	41.4	44.8	27.6	37.9	41.4
Filimonov et al(1957)	54.8	83.9	96.8	62.1	89.7	96.6	48.4	77.4	96.8	66.7	86.7	93.3	50.0	83.3	93.3	55.2	86.2	93.1	58.6	86.2	96.6
Chisholm & Laird(1958)	12.9	22.6	25.8	13.8	24.1	27.6	12.9	19.4	22.6	10.0	16.7	20.0	10.0	13.3	20.0	10.3	13.8	24.1	13.8	17.2	24.1
Flanigan(1958)	6.5	12.9	29.0	10.3	20.7	34.5	6.5	19.4	35.5	3.3	13.3	30.0	0.0	13.3	33.3	6.9	27.6	41.4	10.3	17.2	34.5
Dimentiev et al <sup>2</sup>	22.6	38.7	51.6	27.6	37.9	51.7	25.8	35.5	48.4	23.3	43.3	50.0	26.7	40.0	46.7	27.6	37.9	51.7	27.6	34.5	51.7
Hoogendroon(1959)	0.0	12.9	29.0	0.0	17.2	37.9	3.2	16.1	35.5	0.0	13.3	36.7	0.0	10.0	43.3	0.0	13.8	41.4	0.0	13.8	34.5
Bankoff(1960)	0.0	3.2	32.3	0.0	13.8	34.5	0.0	12.9	51.6	13.3	20.0	56.7	13.3	23.3	56.7	17.2	24.1	51.7	10.3	17.2	41.4
Fauske(1961)	19.4	22.6	29.0	20.7	24.1	27.6	19.4	22.6	25.8	13.3	30.0	36.7	13.3	30.0	33.3	10.3	27.6	31.0	17.2	27.6	31.0
Wilson et al(1961,1965)	12.9	25.8	29.0	3.4	20.7	24.1	6.5	22.6	25.8	10.0	30.0	30.0	10.0	33.3	40.0	3.4	27.6	41.4	6.9	27.6	37.9
Hughmark(1962)	45.2	90.3	96.8	48.3	86.2	96.6	58.1	90.3	100.0	50.0	90.0	100.0	43.3	86.7	96.7	41.4	86.2	93.1	41.4	86.2	93.1
Nicklin et al(1962)	6.5	58.1	87.1	24.1	58.6	89.7	32.3	67.7	93.5	50.0	70.0	93.3	50.0	66.7	90.0	44.8	69.0	82.8	41.4	62.1	86.2
Nishino & Yamazaki(1963)	32.3	48.4	58.1	34.5	48.3	55.2	29.0	51.6	58.1	40.0	56.7	70.0	33.3	53.3	66.7	37.9	55.2	65.5	41.4	48.3	65.5
Fujie(1964)	32.3	51.6	83.9	31.0	55.2	72.4	25.8	45.2	64.5	23.3	43.3	53.3	20.0	40.0	53.3	24.1	34.5	58.6	31.0	48.3	58.6
Kowalczewski(1964)	38.7	45.2	54.8	31.0	44.8	51.7	29.0	48.4	51.6	40.0	50.0	60.0	26.7	50.0	56.7	27.6	51.7	58.6	37.9	51.7	58.6
Thom(1964)	35.5	51.6	61.3	34.5	44.8	58.6	29.0	45.2	48.4	26.7	46.7	56.7	26.7	43.3	50.0	31.0	41.4	55.2	34.5	44.8	55.2
Zivi(1964)	35.5	41.9	48.4	24.1	37.9	51.7	22.6	38.7	48.4	23.3	43.3	53.3	23.3	30.0	50.0	20.7	41.4	51.7	24.1	41.4	51.7
Hughmark(1965)	41.9	74.2	93.5	48.3	62.1	93.1	61.3	74.2	93.5	50.0	73.3	93.3	46.7	63.3	90.0	37.9	58.6	86.2	41.4	62.1	86.2
Neal & Bankoff(1965)	12.9	22.6	25.8	10.3	24.1	27.6	9.7	22.6	29.0	10.0	16.7	26.7	10.0	20.0	23.3	10.3	20.7	31.0	6.9	13.8	20.7
Turner & Wallis(1965)	6.5	12.9	19.4	10.3	13.8	20.7	9.7	16.1	19.4	13.3	16.7	20.0	13.3	16.7	20.0	13.8	17.2	20.7	13.8	17.2	24.1
Baroczy(1966)	32.3	41.9	54.8	31.0	41.4	51.7	29.0	45.2	51.6	30.0	53.3	60.0	26.7	43.3	56.7	27.6	48.3	55.2	37.9	48.3	55.2
Guzhov et al(1967)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Kutucuglu <sup>3</sup>	29.0	48.4	48.4	31.0	34.5	48.3	32.3	41.9	48.4	23.3	40.0	53.3	26.7	40.0	43.3	27.6	41.4	44.8	27.6	41.4	44.8
Smith(1969)	58.1	93.5	100.0	58.6	89.7	93.1	74.2	90.3	100.0	46.7	90.0	96.7	40.0	76.7	86.7	51.7	69.0	86.2	34.5	79.3	89.7
Wallis(1969)	58.1	80.6	87.1	55.2	82.8	86.2	67.7	80.6	83.9	46.7	76.7	86.7	43.3	66.7	86.7	41.4	69.0	86.2	48.3	75.9	93.1



Table 11. Percentage of data points correctly predicted for Beggs (1972) Inclined data (contd.)

Correlation / Inclination Angle	5 Degree data			10 Degree data			15 Degree data			20 Degree data			35 Degree data			55 Degree data			75 Degree data		
	Total Points 31			Total Points 29			Total Points 31			Total Points 30			Total Points 30			Total Points 29			Total Points 29		
	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%
Percentage within																					
Gregory & Scott(1969)	54.8	74.2	100.0	41.4	75.9	100.0	51.6	80.6	100.0	50.0	73.3	100.0	43.3	60.0	90.0	37.9	62.1	86.2	41.4	69.0	100.0
Premoli et al(1970)	61.3	83.9	96.8	58.6	89.7	93.1	61.3	93.5	100.0	66.7	90.0	100.0	56.7	83.3	93.3	58.6	75.9	93.1	62.1	75.9	96.6
Rouhani I(1970) <sup>4</sup>	29.0	83.9	93.5	48.3	86.2	96.6	58.1	87.1	100.0	66.7	86.7	100.0	66.7	86.7	96.7	58.6	89.7	89.7	62.1	86.2	93.1
Rouhani II(1970) <sup>4</sup>	29.0	58.1	74.2	31.0	62.1	86.2	25.8	74.2	87.1	33.3	63.3	93.3	40.0	70.0	90.0	27.6	72.4	96.6	34.5	65.5	93.1
Bonnecaze et al(1971)	6.5	58.1	87.1	24.1	58.6	89.7	32.3	67.7	93.5	50.0	70.0	93.3	50.0	66.7	90.0	44.8	69.0	82.8	41.4	62.1	86.2
Dix(1971)	45.2	77.4	83.9	55.2	82.8	89.7	64.5	83.9	87.1	66.7	83.3	93.3	76.7	93.3	96.7	75.9	96.6	96.6	75.9	89.7	93.1
Beggs(1972)	83.9	87.1	90.3	65.5	89.7	93.1	67.7	77.4	87.1	46.7	66.7	90.0	43.3	66.7	90.0	41.4	65.5	86.2	55.2	72.4	96.6
Chisholm(1973)	80.6	93.5	100.0	69.0	93.1	100.0	74.2	93.5	100.0	56.7	93.3	100.0	40.0	70.0	100.0	51.7	69.0	93.1	48.3	86.2	100.0
Loscher & Reinhardt(1973)	22.6	25.8	41.9	24.1	31.0	37.9	22.6	29.0	32.3	20.0	30.0	50.0	26.7	30.0	43.3	24.1	31.0	44.8	24.1	31.0	44.8
Mattar & Gregory(1974)	0.0	0.0	3.2	0.0	0.0	0.0	0.0	0.0	6.5	0.0	0.0	6.7	0.0	0.0	6.7	0.0	0.0	6.9	0.0	0.0	3.4
Moussali <sup>5</sup>	29.0	45.2	67.7	27.6	48.3	58.6	22.6	38.7	48.4	23.3	30.0	43.3	16.7	30.0	43.3	17.2	34.5	44.8	17.2	41.4	51.7
Greskovich & Cooper(1975)	61.3	80.6	96.8	48.3	72.4	96.6	45.2	64.5	96.8	30.0	53.3	93.3	23.3	43.3	90.0	20.7	55.2	86.2	27.6	55.2	86.2
Madsen(1975)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Mukherjee(1979)	71.0	77.4	90.3	65.5	86.2	96.6	61.3	83.9	90.3	60.0	93.3	93.3	70.0	93.3	96.7	69.0	89.7	96.6	58.6	93.1	96.6
Gardner-1(1980)	16.1	29.0	48.4	20.7	37.9	51.7	16.1	25.8	45.2	10.0	20.0	43.3	10.0	30.0	46.7	20.7	37.9	55.2	20.7	31.0	51.7
Gardner-2(1980)	22.6	38.7	45.2	24.1	37.9	44.8	12.9	32.3	41.9	20.0	36.7	50.0	13.3	36.7	53.3	17.2	37.9	55.2	10.3	44.8	55.2
Sun et al(1980)	6.5	38.7	74.2	6.9	55.2	75.9	22.6	58.1	77.4	33.3	66.7	86.7	43.3	66.7	80.0	41.4	62.1	79.3	31.0	55.2	75.9
Jowitt(1981)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.3	0.0	0.0	13.3	0.0	0.0	13.8	0.0	0.0	13.8
Chisholm(1983) - Armand(1946)	77.4	93.5	100.0	62.1	93.1	100.0	74.2	100.0	100.0	63.3	93.3	100.0	50.0	73.3	100.0	55.2	69.0	100.0	55.2	86.2	96.6
Spedding & Chen(1984)	32.3	41.9	58.1	31.0	37.9	58.6	22.6	41.9	58.1	30.0	53.3	60.0	26.7	46.7	63.3	20.7	48.3	58.6	27.6	51.7	55.2
Bestion(1985)	25.8	41.9	54.8	31.0	41.4	62.1	29.0	45.2	58.1	33.3	43.3	63.3	36.7	50.0	63.3	34.5	51.7	69.0	31.0	44.8	62.1
Tandon et al(1985)	29.0	41.9	58.1	31.0	44.8	55.2	29.0	45.2	58.1	40.0	53.3	63.3	33.3	50.0	63.3	24.1	51.7	65.5	31.0	51.7	62.1
Chen(1986)	35.5	61.3	80.6	41.4	55.2	75.9	22.6	41.9	71.0	20.0	43.3	60.0	26.7	43.3	60.0	24.1	48.3	62.1	20.7	44.8	69.0
Hammersma & Hart(1987)	12.9	25.8	58.1	13.8	24.1	27.6	12.9	22.6	22.6	10.0	16.7	20.0	10.0	13.3	20.0	13.8	17.2	24.1	13.8	17.2	24.1
Kawaji et al(1987)	32.3	45.2	51.6	31.0	41.4	55.2	22.6	38.7	54.8	30.0	53.3	66.7	26.7	50.0	66.7	20.7	58.6	69.0	27.6	48.3	58.6
Minami & Brill(1987)	64.5	83.9	90.3	72.4	86.2	93.1	64.5	80.6	90.3	70.0	90.0	93.3	66.7	93.3	96.7	65.5	89.7	96.6	65.5	89.7	96.6
El-Boher et al(1988)	12.9	67.7	80.6	27.6	72.4	86.2	38.7	67.7	80.6	46.7	86.7	90.0	53.3	93.3	93.3	44.8	86.2	89.7	51.7	75.9	86.2
Hart et al(1989)	32.3	38.7	45.2	31.0	41.4	51.7	29.0	38.7	45.2	36.7	46.7	56.7	26.7	40.0	50.0	27.6	37.9	55.2	24.1	41.4	55.2

Table 11. Percentage of data points correctly predicted for Beggs (1972) Inclined data (contd.)

Correlation / Inclination Angle	5 Degree data			10 Degree data			15 Degree data			20 Degree data			35 Degree data			55 Degree data			75 Degree data		
	Total Points 31			Total Points 29			Total Points 31			Total Points 30			Total Points 30			Total Points 29			Total Points 29		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%
Kokal & Stanislav(1989)	6.5	58.1	87.1	24.1	58.6	89.7	32.3	67.7	93.5	50.0	70.0	93.3	50.0	66.7	90.0	44.8	69.0	82.8	41.4	62.1	86.2
Spedding et al(1989)	29.0	38.7	51.6	31.0	41.4	55.2	22.6	38.7	51.6	30.0	53.3	56.7	26.7	46.7	53.3	20.7	48.3	55.2	27.6	51.7	55.2
Toshiba(1989)	35.5	83.9	93.5	44.8	89.7	96.6	51.6	80.6	93.5	56.7	93.3	96.7	70.0	90.0	93.3	62.1	93.1	93.1	55.2	86.2	93.1
Huq & Loth(1992)	54.8	83.9	96.8	41.4	86.2	93.1	58.1	87.1	100.0	46.7	86.7	96.7	40.0	76.7	86.7	37.9	79.3	82.8	34.5	72.4	86.2
Inoue et al(1993)	32.3	54.8	74.2	37.9	62.1	82.8	35.5	64.5	77.4	40.0	76.7	86.7	43.3	76.7	86.7	44.8	79.3	89.7	37.9	75.9	86.2
Czop et al(1994)	6.5	29.0	51.6	13.8	24.1	58.6	16.1	32.3	54.8	30.0	43.3	63.3	20.0	50.0	66.7	20.7	48.3	62.1	24.1	41.4	55.2
AbdulMajeed(1996)	35.5	58.1	64.5	31.0	58.6	65.5	25.8	51.6	61.3	26.7	56.7	70.0	33.3	53.3	66.7	37.9	51.7	65.5	37.9	62.1	69.0
Maier and Coddington(1997)	22.6	38.7	61.3	24.1	44.8	55.2	22.6	41.9	51.6	30.0	56.7	63.3	40.0	50.0	60.0	34.5	55.2	58.6	34.5	55.2	55.2
Petalaz & Aziz(1997)	6.5	9.7	19.4	6.9	20.7	24.1	9.7	16.1	19.4	3.3	13.3	16.7	6.7	16.7	26.7	3.4	20.7	24.1	6.9	20.7	24.1
Gomez et al(2000)	6.5	6.5	6.5	0.0	3.4	3.4	0.0	3.2	6.5	0.0	0.0	0.0	0.0	0.0	3.3	0.0	3.4	3.4	3.4	3.4	3.4
Zhao et al(2000)	0.0	6.5	9.7	0.0	6.9	10.3	0.0	6.5	9.7	0.0	6.7	10.0	0.0	6.7	13.3	0.0	6.9	10.3	0.0	6.9	10.3
Graham et al(2001)	45.2	67.7	83.9	44.8	65.5	93.1	41.9	80.6	93.5	46.7	90.0	93.3	50.0	83.3	93.3	48.3	86.2	89.7	41.4	82.8	93.1

correlation as reported by Leung (2005), <sup>2,5</sup> correlations as reported by Kataoka and Ishii (1987), <sup>3</sup> correlation as reported by Isbin and Biddle (1979), <sup>4</sup> Rouhani and Axelsson(1970) correlations I and II.

Table 12. Total Number and percentage of data points correctly predicted for the inclined experimental data of Beggs(1972)

Correlation / Inclination Angle	Total data points correctly predicted for the whole database			Percentage of data points correctly predicted for the whole database		
	Total Points 209			Total Points 209		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%
Homogeneous	31	59	82	14.8	28.2	39.2
Armand(1946)	101	144	190	48.3	68.9	90.9
Armand - Massina <sup>1</sup>	122	179	202	58.4	85.6	96.7
Lockhart & Martinelli(1949)	102	167	191	48.8	79.9	91.4
Sterman(1956)	43	76	83	20.6	36.4	39.7
Filimonov et al(1957)	118	177	199	56.5	84.7	95.2
Chisholm & Laird(1958)	25	38	49	12.0	18.2	23.4
Flanigan(1958)	13	37	71	6.2	17.7	34.0
Dimentiev et al <sup>2</sup>	54	80	105	25.8	38.3	50.2
Hoogendroon(1959)	1	29	77	0.5	13.9	36.8
Bankoff(1960)	16	34	97	7.7	16.3	46.4
Fauske(1961)	34	55	64	16.3	26.3	30.6
Wilson et al(1961,1965)	16	56	68	7.7	26.8	32.5
Hughmark(1962)	98	184	202	46.9	88.0	96.7
Nicklin et al(1962)	74	135	186	35.4	64.6	89.0
Nishino & Yamazaki(1963)	74	108	131	35.4	51.7	62.7
Fujie(1964)	56	95	133	26.8	45.5	63.6
Kowalczewski(1964)	69	102	117	33.0	48.8	56.0
Thom(1964)	65	95	115	31.1	45.5	55.0
Zivi(1964)	52	82	106	24.9	39.2	50.7
Hughmark(1965)	98	140	190	46.9	67.0	90.9
Neal & Bankoff(1965)	21	42	55	10.0	20.1	26.3
Turner & Wallis(1965)	24	33	43	11.5	15.8	20.6
Baroczy(1966)	64	96	115	30.6	45.9	55.0
Guzhov et al(1967)	0	0	0	0.0	0.0	0.0
Kutucuglu <sup>3</sup>	59	86	99	28.2	41.1	47.4
Smith(1969)	109	176	195	52.2	84.2	93.3
Wallis(1969)	108	159	182	51.7	76.1	87.1
Gregory & Scott(1969)	96	148	202	45.9	70.8	96.7
Premoli et al(1970)	127	177	201	60.8	84.7	96.2
Rouhani I(1970) <sup>4</sup>	116	181	200	55.5	86.6	95.7
Rouhani II(1970) <sup>4</sup>	66	139	185	31.6	66.5	88.5
Bonnecaze et al(1971)	74	135	186	35.4	64.6	89.0
Dix(1971)	137	181	191	65.6	86.6	91.4
Beggs(1972)	121	157	189	57.9	75.1	90.4
Chisholm(1973)	126	179	207	60.3	85.6	99.0
Loscher & Reinhardt(1973)	49	62	88	23.4	29.7	42.1
Mattar & Gregory(1974)	0	0	10	0.0	0.0	4.8
Moussali <sup>5</sup>	46	80	107	22.0	38.3	51.2
Greskovich & Cooper(1975)	77	127	193	36.8	60.8	92.3
Madsen(1975)	0	0	0	0.0	0.0	0.0
Mukherjee(1979)	136	184	197	65.1	88.0	94.3
Gardner-I(1980)	34	63	102	16.3	30.1	48.8

Table 12. Total Number and percentage of data points correctly predicted for the inclined experimental data of Beggs(1972)(contd.)

Correlation / Inclination Angle	Total data points correctly predicted for the whole database			Percentage of data points correctly predicted for the whole database		
	Total Points 209			Total Points 209		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%
Gardner-2(1980)	36	79	103	17.2	37.8	49.3
Sun et al(1980)	55	120	164	26.3	57.4	78.5
Jowitt(1981)	0	0	13	0.0	0.0	6.2
Chisholm(1983) - Armand(1946)	131	182	208	62.7	87.1	99.5
Spedding & Chen(1984)	57	96	123	27.3	45.9	58.9
Bestion(1985)	66	95	129	31.6	45.5	61.7
Tandon et al(1985)	65	101	127	31.1	48.3	60.8
Chen(1986)	57	101	143	27.3	48.3	68.4
Hamersma & Hart(1987)	26	41	49	12.4	19.6	23.4
Kawaji et al(1987)	57	100	126	27.3	47.8	60.3
Minami & Brill(1987)	140	183	196	67.0	87.6	93.8
El-Boher et al(1988)	82	164	181	39.2	78.5	86.6
Hart et al(1989)	62	85	107	29.7	40.7	51.2
Kokal & Stanislav(1989)	74	135	186	35.4	64.6	89.0
Spedding et al(1989)	56	95	113	26.8	45.5	54.1
Toshiba(1989)	112	184	197	53.6	88.0	94.3
Huq & Loth(1992)	94	171	192	45.0	81.8	91.9
Inoue et al(1993)	81	146	174	38.8	69.9	83.3
Czop et al(1994)	39	80	123	18.7	38.3	58.9
AbdulMajeed(1996)	68	117	138	32.5	56.0	66.0
Maier and Coddington(1997)	62	102	121	29.7	48.8	57.9
Petalaz & Aziz(1997)	13	35	46	6.2	16.7	22.0
Gomez et al(2000)	3	7	8	1.4	3.3	3.8
Zhao et al(2000)	0	14	22	0.0	6.7	10.5
Graham et al(2001)	95	166	191	45.5	79.4	91.4

<sup>1</sup> correlation as reported by Leung (2005), <sup>2,5</sup> correlations as reported by Kataoka and Ishii (1987), <sup>3</sup> correlation as reported by Isbin and Biddle (1979), <sup>4</sup> Rouhani and Axelsson(1970) correlations I and II.

For near horizontal data (2.75 degree) the simple correlation of Lockhart and Martinelli (1949) and Wallis (1969) were the only ones to give a fairly acceptable prediction for the 5% error index. Surprisingly the Lockhart and Martinelli (1949) correlation could not predict the wider error indices acceptably which was not logically expected.

Wallis(1969) and Rouhani I(1970) are two correlations which gave a close to acceptable results for the 10% error index for the near horizontal inclination and

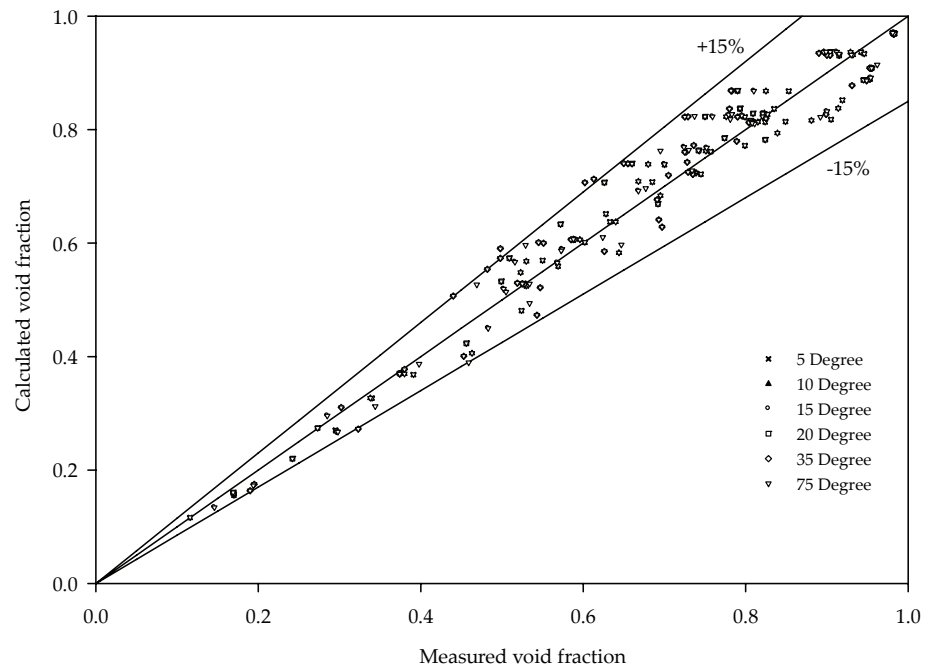


Figure 7. Comparison of Armand Massina (Leung 2005) correlation with measured inclined experimental data of Beggs(1972)

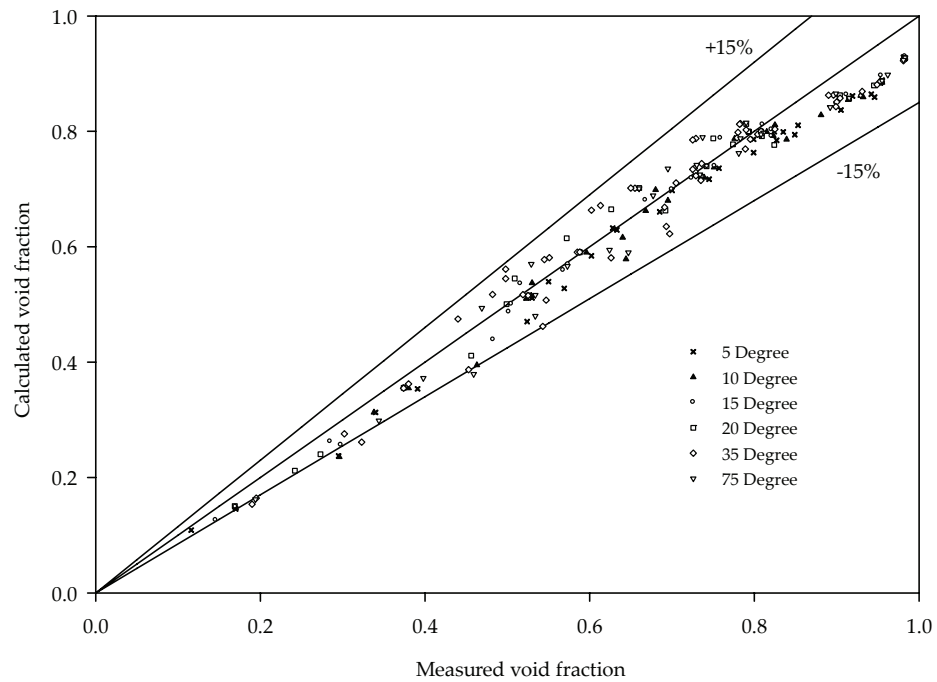


Figure 8. Comparison of Hughmark (1962) correlation with measured inclined experimental data of Beggs(1972)

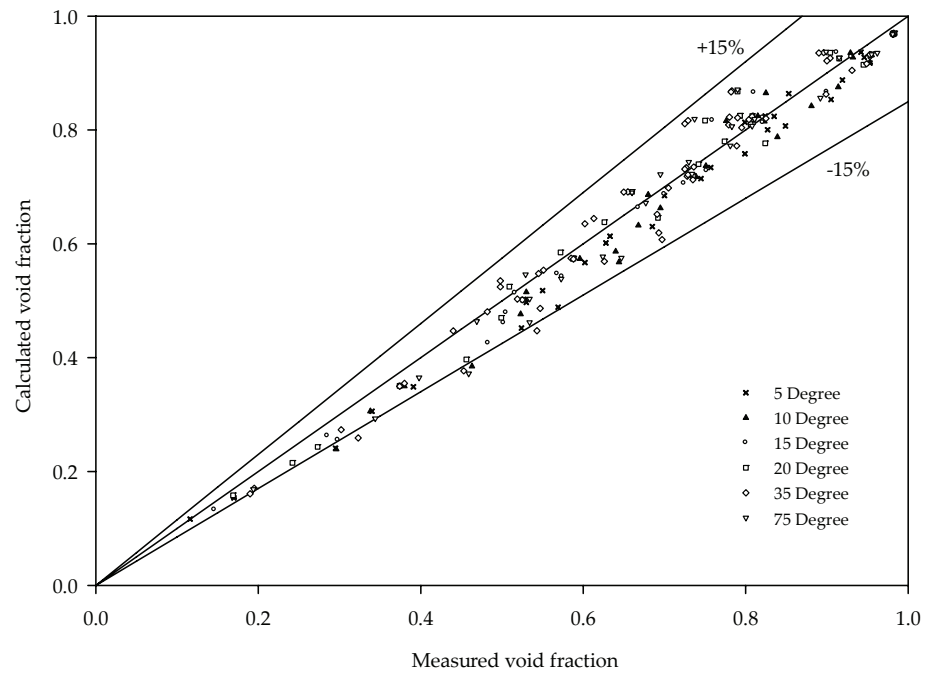


Figure 9. Comparison of Premoli et al. (1970) correlation with measured inclined experimental data of Beggs(1972)

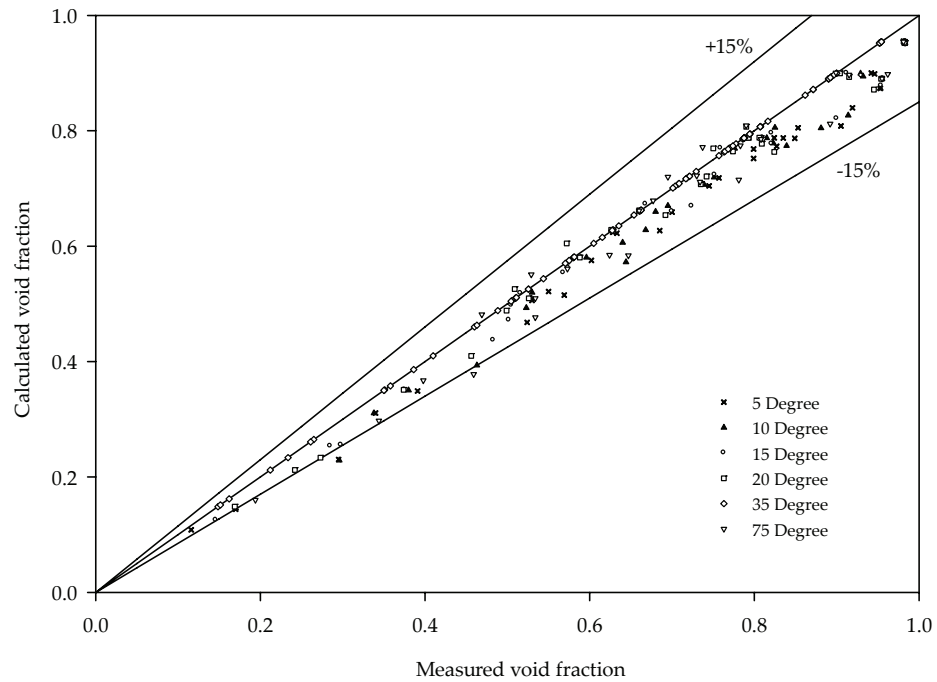


Figure 10. Comparison of Rouhani I (1970) correlation with measured inclined experimental data of Beggs(1972)

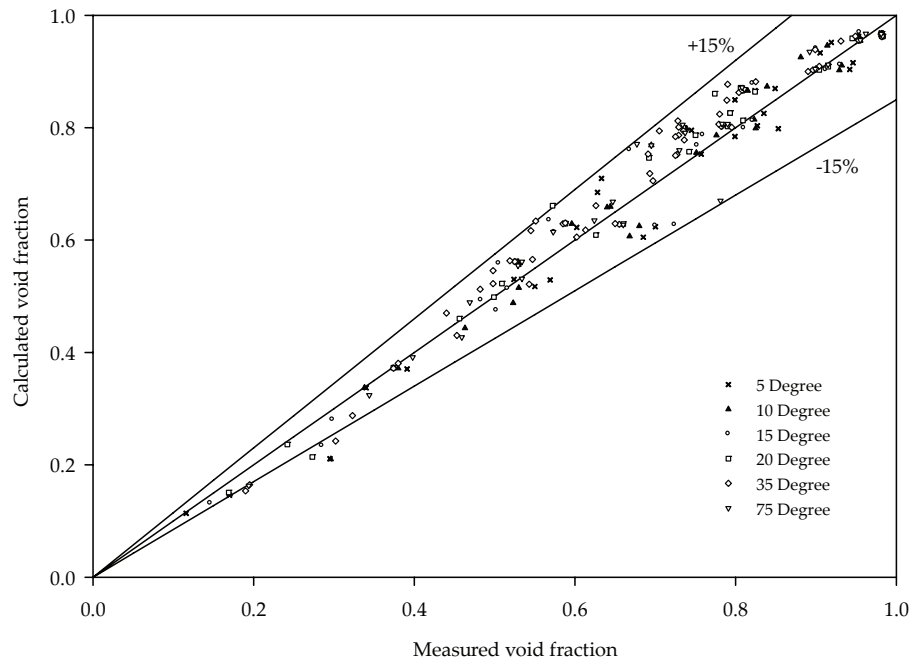


Figure 11. Comparison of Filimonov et al. (1957) correlation with measured inclined experimental data of Beggs(1972)

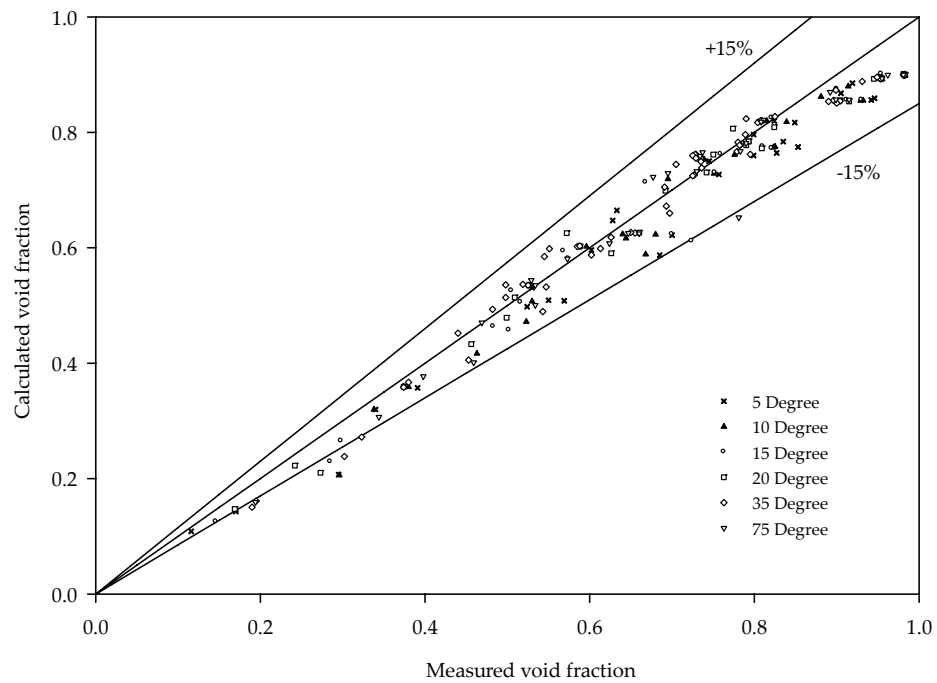


Figure 12. Comparison of Toshiba (1989) correlation with measured inclined experimental data of Beggs(1972)

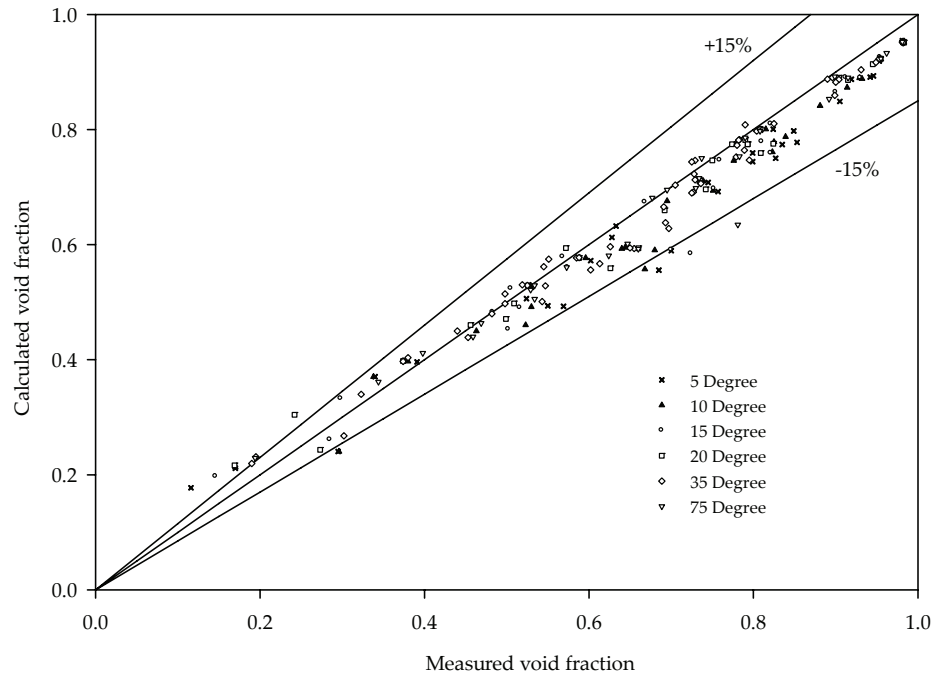


Figure 13. Comparison of Dix (1971) correlation with measured inclined experimental data of Beggs(1972)

Toshiba(1989) for 20.75 degree with none of the correlations acceptably predicting any of the data set at all.

Filimonov et al. (1957), Rouhani I (1970) and Dix (1971) were able to predict the 15% error index for 45 and 75, 2.75 and 20.75 & 75 inclination angles respectively.

The performance of the correlations for all inclination angles for this data set is summarized in Table 15.

The correlation of Toshiba (1989) was the only one that fairly predicted the data set across all inclination angles for error indices of 10 and 15%. The next best performance comes from Dix (1971) followed by Filimonov et al. (1957) and Rouhani I (1970). Though the predictions by Hughmark (1962) and Premoli et al. (1970) fall short of the expected cutoff percentage, we have included a scatter plot to show the void fraction range where they underperformed.



The graphical representation of the capability of the selected correlations with the Spedding and Nguyen(1976) data are given in Figures 15 through 20.

#### **4.2.3 Inclined flow comparison with the data of Mukherjee (1979)**

Mukherjee (1979) reported air - kerosene void fraction data for six pipe inclination angles. The pipe inclinations researched were 5, 20,30,50,70 and 80 Degrees. A total of 444 data points were considered here for making the comparison between the void fraction correlations. The number of data points correctly predicted for each inclination angle by correlations is given in Table 16 while percentage figures of those points are tabulated in Table 17.

Here again the capability of the correlations to predict the data sets within the 5% error index could only average 57.3% of the data points across all inclination angles which is far below what we have set out as an acceptable cut off percentage and far from being applicable to any use by any standard.

The prediction performance of the correlations within the 10% error index has seen a general improvement with the only exception of the 20 degree set for which none of the correlations were able to predict the data on and above the specified 80% cutoff percentage.

Table 13. Number of data points correctly predicted for Spedding & Nguyen (1976)  
Inclined data

Correlation / Inclination Angle	2.75 Degree data			20.75 Degree data			45 Degree data			70 Degree data		
	Total Points 226			Total Points 188			Total Points 196			Total Points 279		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%
Homogeneous	88	115	133	42	55	67	40	53	68	38	53	72
Armand(1946)	32	70	124	45	74	106	30	52	74	47	86	135
Armand - Massina <sup>1</sup>	95	171	184	82	123	136	74	97	103	82	130	151
Lockhart & Martinelli(1949)	163	167	175	97	119	133	79	110	122	99	141	172
Sterman(1956)	89	114	128	44	55	63	42	51	71	39	57	80
Filimonov et al(1957)	103	132	152	71	101	134	95	139	170	105	176	240
Chisholm & Laird(1958)	85	111	123	42	53	61	40	50	65	37	50	67
Flanigan(1958)	83	128	153	56	92	107	52	83	92	68	98	109
Dimentiev et al <sup>2</sup>	78	114	137	61	80	98	47	71	99	54	92	117
Hoogendroon(1959)	88	135	158	59	100	109	58	91	103	83	112	130
Bankoff(1960)	16	38	57	18	41	67	11	24	45	35	72	115
Fauske(1961)	96	112	124	49	60	65	49	63	69	51	74	83
Wilson et al(1961,1965)	20	33	43	16	28	46	18	29	44	18	43	54
Hughmark(1962)	146	174	188	98	134	149	90	101	109	117	143	172
Nicklin et al(1962)	37	80	132	58	95	124	32	67	94	78	136	186
Nishino & Yamazaki(1963)	148	170	180	104	125	129	95	120	134	106	157	179
Fujie(1964)	101	132	157	51	86	106	40	57	82	41	65	102
Kowalczewski(1964)	140	163	169	85	107	108	83	98	107	92	127	143
Thom(1964)	98	150	168	60	90	109	57	91	110	61	105	137
Zivi(1964)	147	157	163	74	88	95	64	81	90	76	98	116
Hughmark(1965)	31	63	109	49	73	101	23	49	65	46	89	127
Neal & Bankoff(1965)	20	31	45	17	33	49	20	38	55	36	61	84
Turner & Wallis(1965)	63	87	104	34	48	54	38	54	61	34	55	68
Baroczy(1966)	153	166	171	85	117	127	75	102	115	93	132	159
Guzhov et al(1967)	3	22	50	1	20	45	1	7	23	0	7	32
Kutucuglu <sup>3</sup>	139	161	167	69	91	98	72	89	97	74	98	117
Smith(1969)	143	171	185	87	126	135	78	102	106	97	131	156
Wallis(1969)	163	182	189	93	119	130	75	102	114	104	139	169
Gregory & Scott(1969)	29	73	146	47	75	113	33	53	84	49	82	140
Premoli et al(1970)	153	174	189	100	136	146	101	121	125	132	176	194
Rouhani I(1970) <sup>4</sup>	95	184	197	97	136	153	71	107	120	104	169	194
Rouhani II(1970) <sup>4</sup>	47	92	132	71	106	141	44	76	105	64	104	153
Bonnecaze et al(1971)	38	80	132	58	95	124	32	67	94	78	136	186
Dix(1971)	150	168	175	114	148	160	112	139	154	165	228	243
Beggs(1972)	120	158	175	57	96	121	54	92	102	60	109	140
Chisholm(1973)	146	172	182	84	121	130	81	97	105	90	124	151
Loscher & Reinhardt(1973)	142	154	156	78	88	99	66	75	81	78	90	102
Mattar & Gregory(1974)	10	23	44	3	23	53	5	21	59	4	22	85
Moussali <sup>5</sup>	89	119	137	46	63	79	40	53	68	38	53	73
Greskovich & Cooper(1975)	110	147	173	51	79	94	41	57	78	41	56	90
Madsen(1975)	20	32	40	14	21	24	12	21	25	8	13	14
Mukherjee(1979)	99	144	172	57	88	131	58	93	113	73	127	173
Gardner-1(1980)	86	127	152	64	97	110	55	81	98	70	103	117
Gardner-2(1980)	92	131	154	60	94	120	56	90	103	79	114	129
Sun et al(1980)	35	73	120	55	88	117	27	62	90	80	139	184
Jowitt(1981)	32	65	157	43	66	111	26	65	120	48	100	165
Chisholm(1983) - Armand(1946)	148	171	182	89	121	130	82	98	106	93	130	152
Spedding & Chen(1984)	155	170	181	94	122	130	88	110	125	104	145	174

Table 13. Number of data points correctly predicted for Spedding & Nguyen (1976)  
Inclined data (contd.)

Correlation / Inclination Angle	2.75 Degree data			20.75 Degree data			45 Degree data			70 Degree data		
	Total Points 226			Total Points 188			Total Points 196			Total Points 279		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%
Bestion(1985)	69	121	144	64	89	98	34	66	84	46	81	97
Tandon et al(1985)	147	164	171	76	92	105	67	86	101	83	110	130
Chen(1986)	143	168	182	84	119	129	80	99	104	96	135	154
Hamersma & Hart(1987)	87	113	124	42	55	63	40	52	65	37	51	68
Kawaji et al(1987)	104	151	167	51	81	97	56	79	92	63	101	118
Minami & Brill(1987)	117	150	177	51	83	119	48	86	104	57	110	150
El-Boher et al(1988)	145	169	184	92	130	149	105	120	124	144	191	206
Hart et al(1989)	150	158	161	85	97	105	66	83	91	84	104	117
Kokal & Stanislav(1989)	36	78	132	58	94	124	32	66	93	77	136	183
Spedding et al(1989)	141	154	160	77	87	93	80	96	102	93	122	138
Toshiba(1989)	56	154	174	60	148	174	63	132	155	123	229	260
Huq & Loth(1992)	152	173	188	99	131	138	84	102	110	107	144	160
Inoue et al(1993)	105	145	156	56	82	113	77	108	133	91	150	192
Czop et al(1994)	25	55	95	35	56	79	23	51	75	55	104	135
AbdulMajeed(1996)	81	130	156	50	92	115	51	85	95	59	114	136
Maier and Coddington(1997)	110	143	158	59	86	109	67	101	111	80	136	156
Petalaz & Aziz(1997)	3	8	18	1	4	9	4	7	11	11	23	33
Gomez et al(2000)	80	118	144	57	82	101	45	70	83	56	84	97
Zhao et al(2000)	63	80	92	33	43	51	35	44	51	27	42	53
Graham et al(2001)	160	169	173	85	108	125	74	99	120	101	139	167

<sup>1</sup> correlation as reported by Leung (2005), <sup>2,5</sup> correlations as reported by Kataoka and Ishii (1987), <sup>3</sup> correlation as reported by Isbin and Biddle (1979), <sup>4</sup> Rouhani and Axelsson(1970) correlations I and II.

Referring to the combined inclined data set of Mukherjee (1979) presented in Table 18, the correlation of Gregory & Scott (1969), Armand – Massina (Leung 2005), Rouhani I (1970) and Hughmark (1962) have performed well on the 15% index and in aggregate across the whole range of inclination angles considered and are worth a recommendation for this data set.

The correlations that fall into the next best category are that of Beggs (1972), Chisholm (1973), Greskovich & Cooper (1975), Chisholm (1983) – Armand (1946), Sun et al. (1980) and Minami & Brill (1987).

The correlations of Premoli et al. (1970), Toshiba (1989), Dix (1971) and Filimonov et al. (1957) which will be shown to have an acceptable performance for the combined

Table 14. Percentage of data points correctly predicted for Spedding & Nguyen (1976) Inclined data

Correlation / Inclination Angle	2.75 Degree data			20.75 Degree data			45 Degree data			70 Degree data		
	Total Points 226			Total Points 188			Total Points 196			Total Points 279		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%
Homogeneous	38.9	50.9	58.8	22.3	29.3	35.6	20.4	27.0	34.7	13.6	19.0	25.8
Armand(1946)	14.2	31.0	54.9	23.9	39.4	56.4	15.3	26.5	37.8	16.8	30.8	48.4
Armand - Massina <sup>1</sup>	42.0	75.7	81.4	43.6	65.4	72.3	37.8	49.5	52.6	29.4	46.6	54.1
Lockhart & Martinelli(1949)	72.1	73.9	77.4	51.6	63.3	70.7	40.3	56.1	62.2	35.5	50.5	61.6
Sterman(1956)	39.4	50.4	56.6	23.4	29.3	33.5	21.4	26.0	36.2	14.0	20.4	28.7
Filimonov et al(1957)	45.6	58.4	67.3	37.8	53.7	71.3	48.5	70.9	86.7	37.6	63.1	86.0
Chisholm & Laird(1958)	37.6	49.1	54.4	22.3	28.2	32.4	20.4	25.5	33.2	13.3	17.9	24.0
Flanigan(1958)	36.7	56.6	67.7	29.8	48.9	56.9	26.5	42.3	46.9	24.4	35.1	39.1
Dimentiev et al <sup>2</sup>	34.5	50.4	60.6	32.4	42.6	52.1	24.0	36.2	50.5	19.4	33.0	41.9
Hoogendroon(1959)	38.9	59.7	69.9	31.4	53.2	58.0	29.6	46.4	52.6	29.7	40.1	46.6
Bankoff(1960)	7.1	16.8	25.2	9.6	21.8	35.6	5.6	12.2	23.0	12.5	25.8	41.2
Fauske(1961)	42.5	49.6	54.9	26.1	31.9	34.6	25.0	32.1	35.2	18.3	26.5	29.7
Wilson et al(1961,1965)	8.8	14.6	19.0	8.5	14.9	24.5	9.2	14.8	22.4	6.5	15.4	19.4
Hughmark(1962)	64.6	77.0	83.2	52.1	71.3	79.3	45.9	51.5	55.6	41.9	51.3	61.6
Nicklin et al(1962)	16.4	35.4	58.4	30.9	50.5	66.0	16.3	34.2	48.0	28.0	48.7	66.7
Nishino & Yamazaki(1963)	65.5	75.2	79.6	55.3	66.5	68.6	48.5	61.2	68.4	38.0	56.3	64.2
Fujie(1964)	44.7	58.4	69.5	27.1	45.7	56.4	20.4	29.1	41.8	14.7	23.3	36.6
Kowalczewski(1964)	61.9	72.1	74.8	45.2	56.9	57.4	42.3	50.0	54.6	33.0	45.5	51.3
Thom(1964)	43.4	66.4	74.3	31.9	47.9	58.0	29.1	46.4	56.1	21.9	37.6	49.1
Zivi(1964)	65.0	69.5	72.1	39.4	46.8	50.5	32.7	41.3	45.9	27.2	35.1	41.6
Hughmark(1965)	13.7	27.9	48.2	26.1	38.8	53.7	11.7	25.0	33.2	16.5	31.9	45.5
Neal & Bankoff(1965)	8.8	13.7	19.9	9.0	17.6	26.1	10.2	19.4	28.1	12.9	21.9	30.1
Turner & Wallis(1965)	27.9	38.5	46.0	18.1	25.5	28.7	19.4	27.6	31.1	12.2	19.7	24.4
Baroczy(1966)	67.7	73.5	75.7	45.2	62.2	67.6	38.3	52.0	58.7	33.3	47.3	57.0
Guzhov et al(1967)	1.3	9.7	22.1	0.5	10.6	23.9	0.5	3.6	11.7	0.0	2.5	11.5
Kutucuglu <sup>3</sup>	61.5	71.2	73.9	36.7	48.4	52.1	36.7	45.4	49.5	26.5	35.1	41.9
Smith(1969)	63.3	75.7	81.9	46.3	67.0	71.8	39.8	52.0	54.1	34.8	47.0	55.9
Wallis(1969)	72.1	80.5	83.6	49.5	63.3	69.1	38.3	52.0	58.2	37.3	49.8	60.6
Gregory & Scott(1969)	12.8	32.3	64.6	25.0	39.9	60.1	16.8	27.0	42.9	17.6	29.4	50.2
Premoli et al(1970)	67.7	77.0	83.6	53.2	72.3	77.7	51.5	61.7	63.8	47.3	63.1	69.5
Rouhani I(1970) <sup>4</sup>	42.0	81.4	87.2	51.6	72.3	81.4	36.2	54.6	61.2	37.3	60.6	69.5
Rouhani II(1970) <sup>4</sup>	20.8	40.7	58.4	37.8	56.4	75.0	22.4	38.8	53.6	22.9	37.3	54.8
Bonnecaze et al(1971)	16.8	35.4	58.4	30.9	50.5	66.0	16.3	34.2	48.0	28.0	48.7	66.7
Dix(1971)	66.4	74.3	77.4	60.6	78.7	85.1	57.1	70.9	78.6	59.1	81.7	87.1
Beggs(1972)	53.1	69.9	77.4	30.3	51.1	64.4	27.6	46.9	52.0	21.5	39.1	50.2
Chisholm(1973)	64.6	76.1	80.5	44.7	64.4	69.1	41.3	49.5	53.6	32.3	44.4	54.1
Loscher & Reinhardt(1973)	62.8	68.1	69.0	41.5	46.8	52.7	33.7	38.3	41.3	28.0	32.3	36.6
Mattar & Gregory(1974)	4.4	10.2	19.5	1.6	12.2	28.2	2.6	10.7	30.1	1.4	7.9	30.5
Moussali <sup>5</sup>	39.4	52.7	60.6	24.5	33.5	42.0	20.4	27.0	34.7	13.6	19.0	26.2
Greskovich & Cooper(1975)	48.7	65.0	76.5	27.1	42.0	50.0	20.9	29.1	39.8	14.7	20.1	32.3
Madsen(1975)	8.8	14.2	17.7	7.4	11.2	12.8	6.1	10.7	12.8	2.9	4.7	5.0
Mukherjee(1979)	43.8	63.7	76.1	30.3	46.8	69.7	29.6	47.4	57.7	26.2	45.5	62.0
Gardner-1(1980)	38.1	56.2	67.3	34.0	51.6	58.5	28.1	41.3	50.0	25.1	36.9	41.9
Gardner-2(1980)	40.7	58.0	68.1	31.9	50.0	63.8	28.6	45.9	52.6	28.3	40.9	46.2
Sun et al(1980)	15.5	32.3	53.1	29.3	46.8	62.2	13.8	31.6	45.9	28.7	49.8	65.9
Jowitt(1981)	14.2	28.8	69.5	22.9	35.1	59.0	13.3	33.2	61.2	17.2	35.8	59.1
Chisholm(1983) - Armand(1946)	65.5	75.7	80.5	47.3	64.4	69.1	41.8	50.0	54.1	33.3	46.6	54.5
Spedding & Chen(1984)	68.6	75.2	80.1	50.0	64.9	69.1	44.9	56.1	63.8	37.3	52.0	62.4

Table14. Percentage of data points correctly predicted for Spedding & Nguyen (1976)  
Inclined data (contd.)

Correlation / Inclination Angle	2.75 Degree data			20.75 Degree data			45 Degree data			70 Degree data		
	Total Points 226			Total Points 188			Total Points 196			Total Points 279		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%
Bestion(1985)	30.5	53.5	63.7	34.0	47.3	52.1	17.3	33.7	42.9	16.5	29.0	34.8
Tandon et al(1985)	65.0	72.6	75.7	40.4	48.9	55.9	34.2	43.9	51.5	29.7	39.4	46.6
Chen(1986)	63.3	74.3	80.5	44.7	63.3	68.6	40.8	50.5	53.1	34.4	48.4	55.2
Hamersma & Hart(1987)	38.5	50.0	54.9	22.3	29.3	33.5	20.4	26.5	33.2	13.3	18.3	24.4
Kawaji et al(1987)	46.0	66.8	73.9	27.1	43.1	51.6	28.6	40.3	46.9	22.6	36.2	42.3
Minami & Brill(1987)	51.8	66.4	78.3	27.1	44.1	63.3	24.5	43.9	53.1	20.4	39.4	53.8
El-Boher et al(1988)	64.2	74.8	81.4	48.9	69.1	79.3	53.6	61.2	63.3	51.6	68.5	73.8
Hart et al(1989)	66.4	69.9	71.2	45.2	51.6	55.9	33.7	42.3	46.4	30.1	37.3	41.9
Kokal & Stanislav(1989)	15.9	34.5	58.4	30.9	50.0	66.0	16.3	33.7	47.4	27.6	48.7	65.6
Spedding et al(1989)	62.4	68.1	70.8	41.0	46.3	49.5	40.8	49.0	52.0	33.3	43.7	49.5
Toshiba(1989)	24.8	68.1	77.0	31.9	78.7	92.6	32.1	67.3	79.1	44.1	82.1	93.2
Huq & Loth(1992)	67.3	76.5	83.2	52.7	69.7	73.4	42.9	52.0	56.1	38.4	51.6	57.3
Inoue et al(1993)	46.5	64.2	69.0	29.8	43.6	60.1	39.3	55.1	67.9	32.6	53.8	68.8
Czop et al(1994)	11.1	24.3	42.0	18.6	29.8	42.0	11.7	26.0	38.3	19.7	37.3	48.4
AbdulMajeed(1996)	35.8	57.5	69.0	26.6	48.9	61.2	26.0	43.4	48.5	21.1	40.9	48.7
Maier and Coddington(1997)	48.7	63.3	69.9	31.4	45.7	58.0	34.2	51.5	56.6	28.7	48.7	55.9
Petalaz & Aziz(1997)	1.3	3.5	8.0	0.5	2.1	4.8	2.0	3.6	5.6	3.9	8.2	11.8
Gomez et al(2000)	35.4	52.2	63.7	30.3	43.6	53.7	23.0	35.7	42.3	20.1	30.1	34.8
Zhao et al(2000)	27.9	35.4	40.7	17.6	22.9	27.1	17.9	22.4	26.0	9.7	15.1	19.0
Graham et al(2001)	70.8	74.8	76.5	45.2	57.4	66.5	37.8	50.5	61.2	36.2	49.8	59.9

<sup>1</sup> correlation as reported by Leung (2005), <sup>2,5</sup> correlations as reported by Kataoka and Ishii (1987), <sup>3</sup> correlation as reported by Isbin and Biddle (1979) , <sup>4</sup> Rouhani and Axelsson(1970) correlations I and II.

inclined database only gave a reasonable average prediction of 75% on the 15% error index for this data set . Once again the scatter plot is used to show where the weaknesses of the top performing correlations lie rather than presenting the correlations that have performed well for this dataset only. These are given in Figures 21 to 27.

Having separately considered each data set for each inclination angle, we have consolidated all the results from all the sources by error index so that we may infer general conclusions on the performance of the correlations. The total number and percentage of the total data points correctly predicted for the combined inclined database for each error indices is reported in Table 19.

Table 15. Total Number and percentage of data points correctly predicted for the inclined experimental data of Spedding & Nguyen (1976)

Correlation / Inclination Angle	Total data points correctly predicted for the whole database			Percentage of data points correctly predicted for the whole database		
	Total Points 889			Total Points 889		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%
Homogeneous	208	276	340	23.4	31.0	38.2
Armand(1946)	154	282	439	17.3	31.7	49.4
Armand - Massina <sup>1</sup>	333	521	574	37.5	58.6	64.6
Lockhart & Martinelli(1949)	438	537	602	49.3	60.4	67.7
Sterman(1956)	214	277	342	24.1	31.2	38.5
Filimonov et al(1957)	374	548	696	42.1	61.6	78.3
Chisholm & Laird(1958)	204	264	316	22.9	29.7	35.5
Flanigan(1958)	259	401	461	29.1	45.1	51.9
Dimentiev et al <sup>2</sup>	240	357	451	27.0	40.2	50.7
Hoogendroon(1959)	288	438	500	32.4	49.3	56.2
Bankoff(1960)	80	175	284	9.0	19.7	31.9
Fauske(1961)	245	309	341	27.6	34.8	38.4
Wilson et al(1961,1965)	72	133	187	8.1	15.0	21.0
Hughmark(1962)	451	552	618	50.7	62.1	69.5
Nicklin et al(1962)	205	378	536	23.1	42.5	60.3
Nishino & Yamazaki(1963)	453	572	622	51.0	64.3	70.0
Fujie(1964)	233	340	447	26.2	38.2	50.3
Kowalczewski(1964)	400	495	527	45.0	55.7	59.3
Thom(1964)	276	436	524	31.0	49.0	58.9
Zivi(1964)	361	424	464	40.6	47.7	52.2
Hughmark(1965)	149	274	402	16.8	30.8	45.2
Neal & Bankoff(1965)	93	163	233	10.5	18.3	26.2
Turner & Wallis(1965)	169	244	287	19.0	27.4	32.3
Baroczy(1966)	406	517	572	45.7	58.2	64.3
Guzhov et al(1967)	5	56	150	0.6	6.3	16.9
Kutucuglu <sup>3</sup>	354	439	479	39.8	49.4	53.9
Smith(1969)	405	530	582	45.6	59.6	65.5
Wallis(1969)	435	542	602	48.9	61.0	67.7
Gregory & Scott(1969)	158	283	483	17.8	31.8	54.3
Premoli et al(1970)	486	607	654	54.7	68.3	73.6
Rouhani I(1970) <sup>4</sup>	367	596	664	41.3	67.0	74.7
Rouhani II(1970) <sup>4</sup>	226	378	531	25.4	42.5	59.7
Bonnecaze et al(1971)	206	378	536	23.2	42.5	60.3
Dix(1971)	541	683	732	60.9	76.8	82.3
Beggs(1972)	291	455	538	32.7	51.2	60.5
Chisholm(1973)	401	514	568	45.1	57.8	63.9
Loscher & Reinhardt(1973)	364	407	438	40.9	45.8	49.3
Mattar & Gregory(1974)	22	89	241	2.5	10.0	27.1
Moussali <sup>5</sup>	213	288	357	24.0	32.4	40.2
Greskovich & Cooper(1975)	243	339	435	27.3	38.1	48.9
Madsen(1975)	54	87	103	6.1	9.8	11.6
Mukherjee(1979)	287	452	589	32.3	50.8	66.3
Gardner-1(1980)	275	408	477	30.9	45.9	53.7

Table 15. Total Number and percentage of data points correctly predicted for the inclined experimental data of Spedding & Nguyen (1976) (contd.)

Correlation / Inclination Angle	Total data points correctly predicted for the whole database			Percentage of data points correctly predicted for the whole database		
	Total Points 889			Total Points 889		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%
Gardner-2(1980)	287	429	506	32.3	48.3	56.9
Sun et al(1980)	197	362	511	22.2	40.7	57.5
Jowitt(1981)	149	296	553	16.8	33.3	62.2
Chisholm(1983) - Armand(1946)	412	520	570	46.3	58.5	64.1
Spedding & Chen(1984)	441	547	610	49.6	61.5	68.6
Bestion(1985)	213	357	423	24.0	40.2	47.6
Tandon et al(1985)	373	452	507	42.0	50.8	57.0
Chen(1986)	403	521	569	45.3	58.6	64.0
Hamersma & Hart(1987)	206	271	320	23.2	30.5	36.0
Kawaji et al(1987)	274	412	474	30.8	46.3	53.3
Minami & Brill(1987)	273	429	550	30.7	48.3	61.9
El-Boher et al(1988)	486	610	663	54.7	68.6	74.6
Hart et al(1989)	385	442	474	43.3	49.7	53.3
Kokal & Stanislav(1989)	203	374	532	22.8	42.1	59.8
Spedding et al(1989)	391	459	493	44.0	51.6	55.5
Toshiba(1989)	302	663	763	34.0	74.6	85.8
Huq & Loth(1992)	442	550	596	49.7	61.9	67.0
Inoue et al(1993)	329	485	594	37.0	54.6	66.8
Czop et al(1994)	138	266	384	15.5	29.9	43.2
AbdulMajeed(1996)	241	421	502	27.1	47.4	56.5
Maier and Coddington(1997)	316	466	534	35.5	52.4	60.1
Petalaz & Aziz(1997)	19	42	71	2.1	4.7	8.0
Gomez et al(2000)	238	354	425	26.8	39.8	47.8
Zhao et al(2000)	158	209	247	17.8	23.5	27.8
Graham et al(2001)	420	515	585	47.2	57.9	65.8

<sup>1</sup> correlation as reported by Leung (2005), <sup>2,5</sup> correlations as reported by Kataoka and Ishii (1987), <sup>3</sup> correlation as reported by Isbin and Biddle (1979), <sup>4</sup> Rouhani and Axelsson(1970) correlations I and II.

For the range of inclination angles considered from all sources, none of the correlations gave acceptable results within the 5% error index. The only correlation worth consideration being that of Minami and Brill (1987) which predicted 67% of the inclined data set of Beggs (1972).

The results of the wider error bands are a little bit encouraging where quite a number of correlations were able to predict the data of Beggs (1972) and Mukherjee (1979).

The data of Spedding and Nguyen (1976) yielded only to the correlation of Toshiba

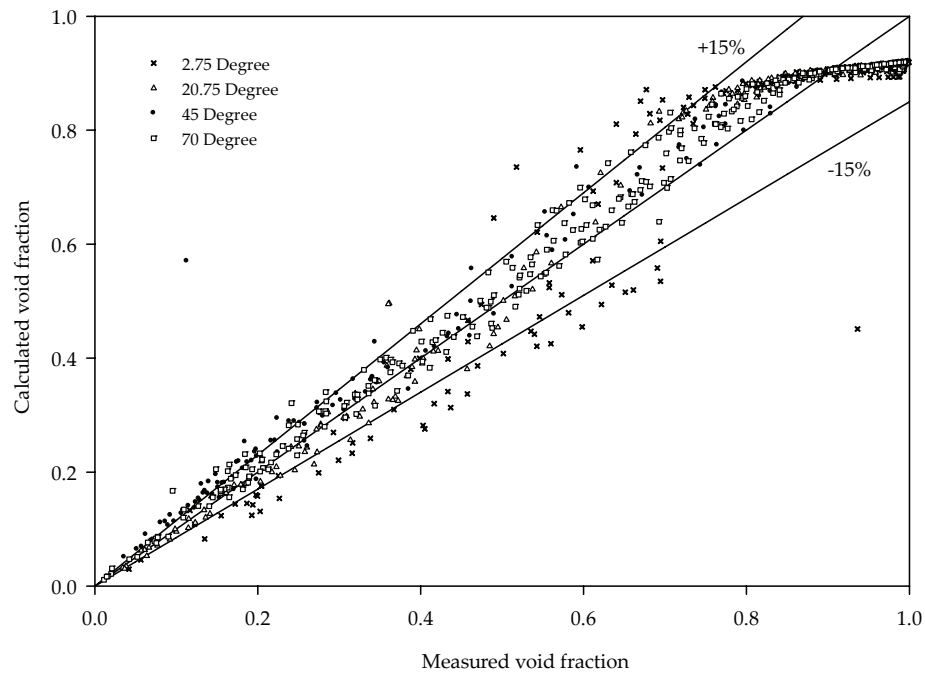


Figure 14. Comparison of Toshiba (1989) correlation with measured inclined experimental data of Spedding & Nguyen (1976)

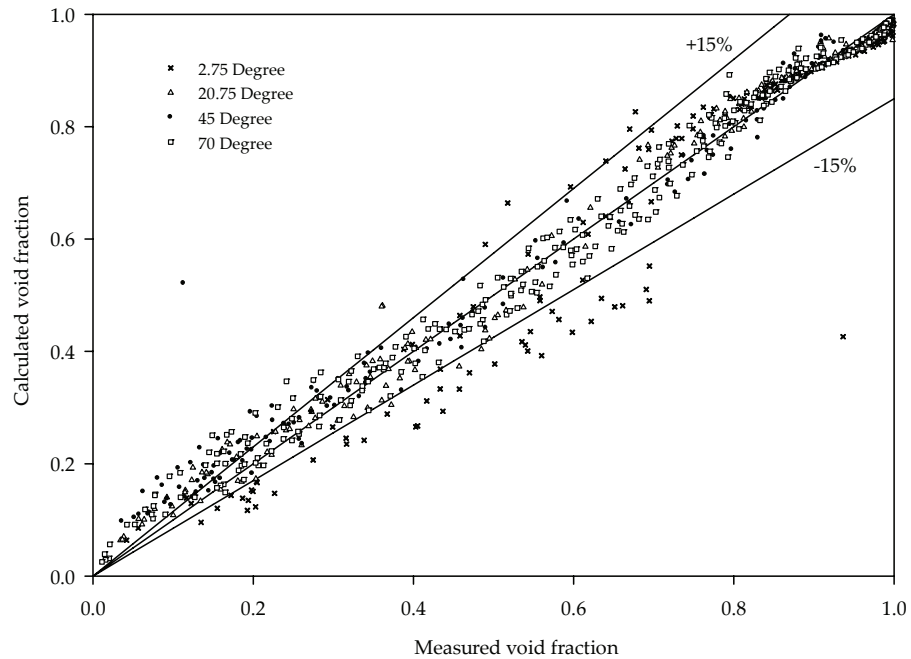


Figure 15. Comparison of Dix (1971) correlation with measured inclined experimental data of Spedding & Nguyen (1976)



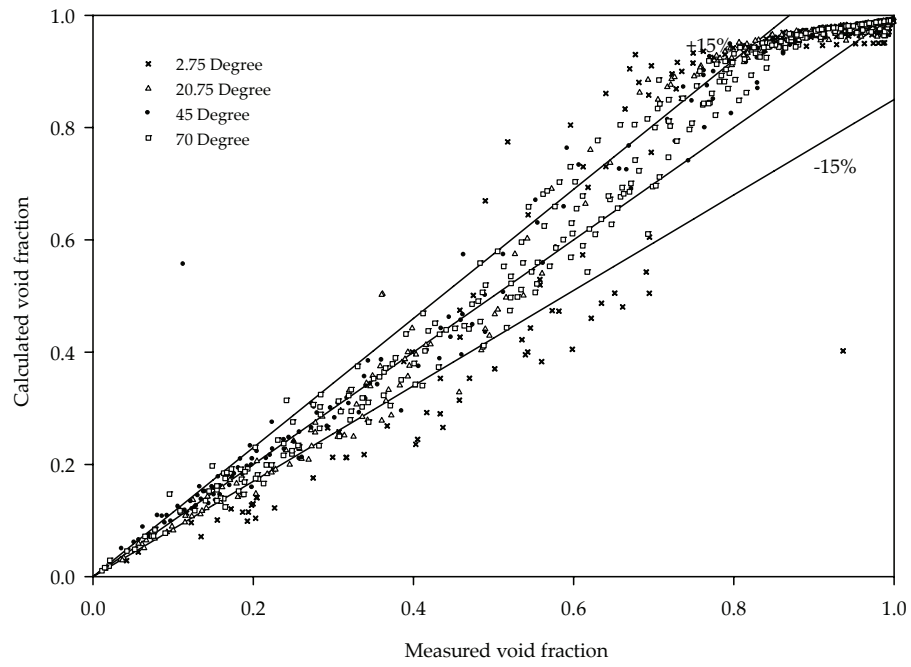


Figure 16. Comparison of Filimonov et al. (1957) correlation with measured inclined experimental data of Spedding & Nguyen (1976)

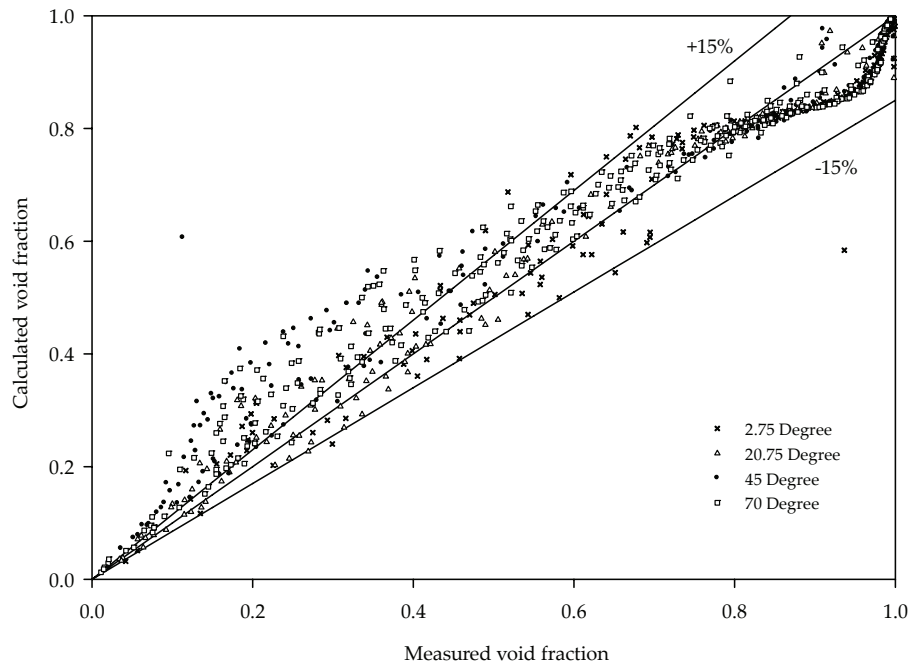


Figure 17. Comparison of Rouhani I (1970) correlation with measured inclined experimental data of Spedding & Nguyen (1976)

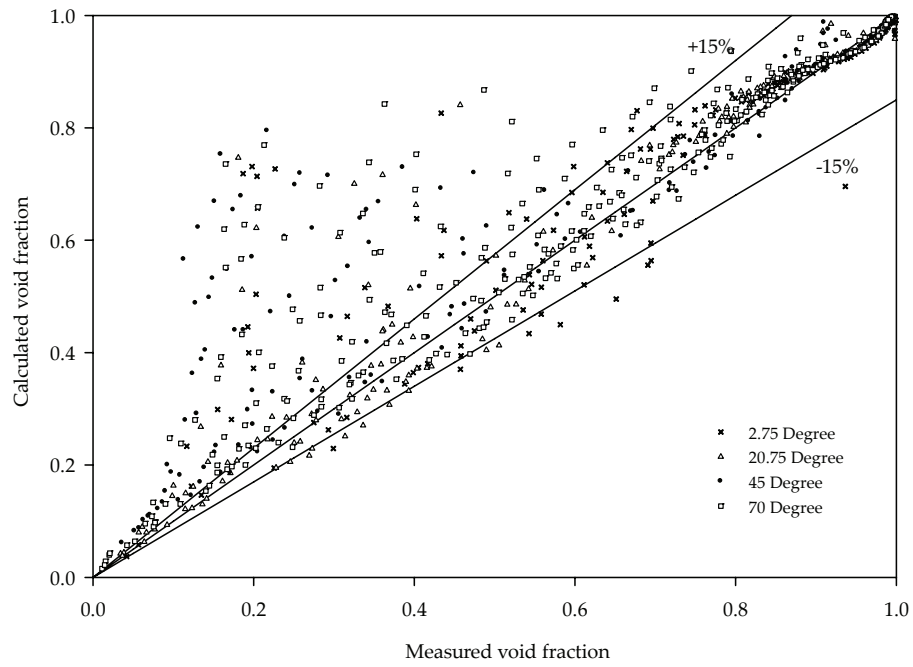


Figure 18. Comparison of Premoli et al. (1970) correlation with measured inclined experimental data of Spedding & Nguyen (1976)

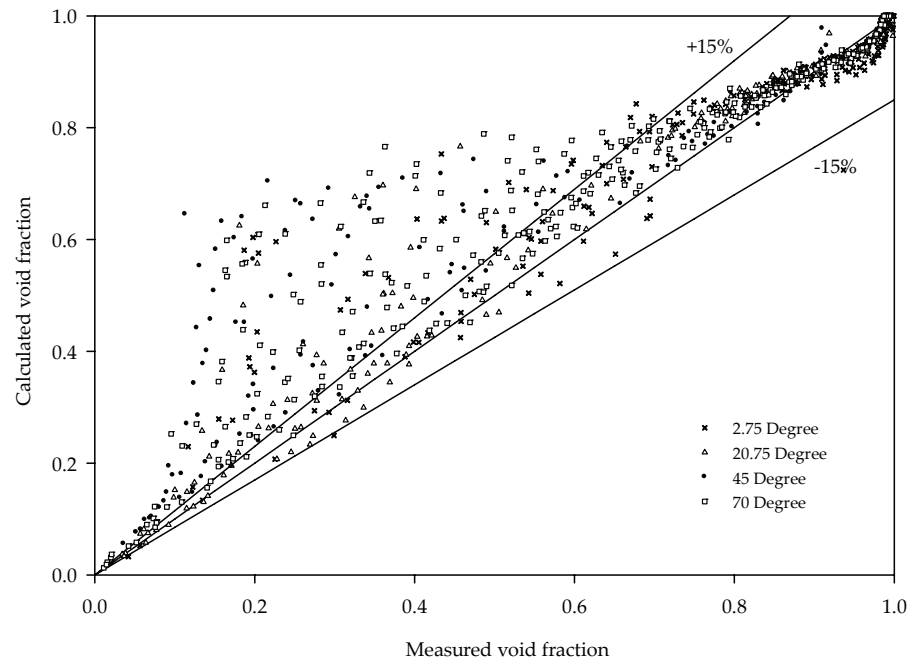


Figure 19. Comparison of Hughmark (1962) correlation with measured inclined experimental data of Spedding & Nguyen (1976)

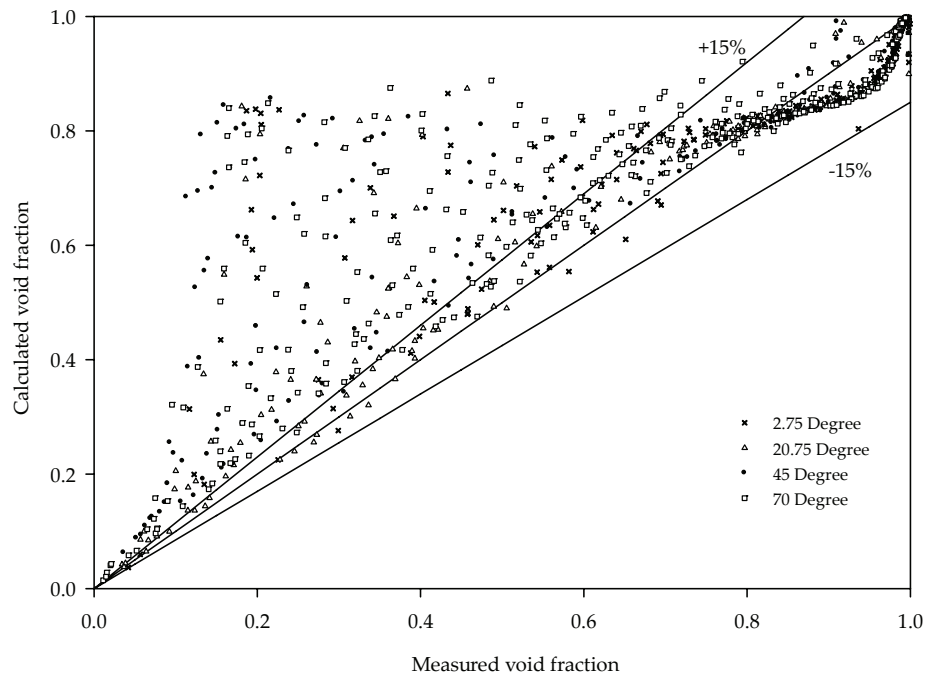


Figure 20. Comparison of Armand Massina (Leung 2005) correlation with measured inclined experimental data of Spedding & Nguyen (1976)

(1989) and Dix (1971) where the latter correlation falls short of the set cutoff percentage by a slight margin.

It is our conclusion that the poor performance of the correlations within the narrow error index (5%) may not be due to the weaknesses of the correlations alone but the accuracy of the data sets which for most of the cases was not explicitly reported and should also be questioned. This observation is not confined to the current data set alone but equally applies to all the data sets used in the horizontal, inclined and the subsequent vertical comparisons.

As was in the horizontal comparison, a simple scatter plot of the measured versus calculated void fraction for six correlations that fairly predicted the whole inclined set is given in Figures 28 through 34. The correlations worth recommendation for this

data set are that of Toshiba(1989), Dix(1971), Rouhani I(1970), Filimonov et al. (1957) ,Premoli et al. (1970) and Hughmark(1962) in order of decreasing accuracy.

The correlation of Toshiba (1989) and Dix (1971) were seen not only to give a very good prediction but also the most consistent performance across the whole void fraction range. The slight under prediction by Dix (1971) and Toshiba (1989) for void fractions less than 0.8 is quite acceptable. Rouhani I (1970) has a moderate over prediction for void fraction less than 0.6 while the correlations of Premoli et al. (1970) and Hughmark (1962) have an excessive over prediction for void fractions less than 0.8. The range involved in this case is very wide making a general recommendation of these two correlations a bit controversial despite their high percentage of data points correctly predicted within the 15% error index especially for the correlation of Premoli et al. (1970).

The fifth ranked Filimonov et al. (1957) correlation has a fairly acceptable scatter across the whole range which is much better than the performance by Premoli et al. (1970) which is ranked fourth in terms of the aggregate data points correctly predicted for the inclined data set.

Splitting the consolidated results by mixture type, we have analyzed the performance of the correlations for the air-water and air-kerosene mixtures separately. For the wider 15% error percentage index, the correlation of Toshiba (1989) predicted 87.4% of the combined data (1098 points) of Beggs (1972) and Spedding and Nguyen (1976) while Dix (1971) and Filimonov et al. (1957) gave 84.1 and 81.5%, respectively.

Table 16. Number of data points correctly predicted for Mukherjee (1979) Inclined data

Correlation / Inclination Angle	5 Degree data Total Points 57			20 Degree data Total Points 74			30 Degree data Total Points 74			50 Degree data Total Points 67			70 Degree data Total Points 80			80 Degree data Total Points 92		
	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%
	Percentage within																	
Homogeneous	16	32	37	25	42	50	16	37	50	10	27	37	21	37	52	18	34	50
Armand(1946)	16	31	52	15	30	60	20	34	67	13	36	50	16	40	69	22	49	72
Armand - Massina <sup>1</sup>	23	42	54	20	45	65	28	47	68	17	43	51	27	59	70	39	72	79
Lockhart & Martinelli(1949)	22	33	43	23	40	47	17	31	49	16	32	42	28	56	67	34	63	78
Sterman(1956)	6	20	29	10	26	35	8	21	31	6	12	22	9	20	30	15	36	42
Filimonov et al(1957)	31	46	47	37	58	64	45	58	60	31	44	47	42	54	55	50	64	71
Chisholm & Laird(1958)	6	18	27	10	24	30	4	13	20	5	10	19	6	19	29	13	25	35
Flanigan(1958)	7	16	23	12	26	37	15	32	41	3	18	30	4	12	25	2	15	27
Dimentiev et al <sup>2</sup>	8	19	29	11	26	37	12	25	34	11	22	33	16	37	46	15	35	42
Hoogendroon(1959)	6	18	25	6	27	41	5	29	46	5	19	31	4	11	32	14	29	42
Bankoff(1960)	0	2	12	3	10	20	3	5	11	4	10	15	5	10	19	4	12	20
Fauske(1961)	10	13	17	10	19	22	8	10	12	11	16	20	11	17	20	22	28	31
Wilson et al(1961,1965)	7	15	21	10	25	32	11	27	39	10	22	27	10	16	25	12	18	20
Hughmark(1962)	17	39	53	21	45	60	23	46	66	19	39	54	19	51	62	32	69	81
Nicklin et al(1962)	4	21	38	11	25	50	6	28	58	11	28	53	9	26	53	21	43	66
Nishino & Yamazaki(1963)	21	27	39	19	32	40	15	24	32	15	26	32	14	33	45	22	41	56
Fujie(1964)	22	40	47	30	50	61	30	51	68	18	32	45	16	52	66	23	52	71
Kowalczewski(1964)	23	36	42	27	36	40	17	31	36	15	30	34	28	41	43	28	50	56
Thom(1964)	21	34	36	25	35	39	17	28	32	17	29	33	27	38	41	31	40	48
Zivi(1964)	21	30	35	23	30	32	14	17	22	12	21	29	21	32	38	29	36	49
Hughmark(1965)	16	24	44	12	28	51	16	32	65	12	27	46	15	36	61	22	46	66
Neal & Bankoff(1965)	11	14	16	10	16	22	7	9	11	4	5	10	12	24	25	5	10	12
Turner & Wallis(1965)	1	9	12	6	12	16	3	7	8	1	8	13	2	6	14	6	15	25
Baroczy(1966)	20	27	37	20	28	37	14	20	29	13	22	31	17	30	40	20	37	50
Guzhov et al(1967)	0	0	0	0	0	1	0	0	0	0	0	4	0	0	1	0	0	0
Kutucuglu <sup>3</sup>	16	28	34	18	29	40	14	22	25	13	23	32	30	40	45	26	36	46

Table 16. Number of data points correctly predicted for Mukherjee (1979) Inclined data (contd.)

Correlation / Inclination Angle	5 Degree data			20 Degree data			30 Degree data			50 Degree data			70 Degree data			80 Degree data		
	Total Points 57			Total Points 74			Total Points 74			Total Points 67			Total Points 80			Total Points 92		
	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%
Percentage within	24	42	48	29	44	54	25	49	57	17	36	49	35	58	68	40	68	79
Smith(1969)																		
Wallis(1969)	22	29	43	24	36	48	18	31	52	16	31	43	25	53	64	25	52	70
Gregory & Scott(1969)	16	33	55	13	30	67	21	38	66	13	36	51	16	41	70	26	52	80
Premoli et al(1970)	31	42	47	42	51	60	43	56	61	20	42	54	42	57	62	43	58	72
Rouhani I(1970) <sup>4</sup>	13	35	52	19	40	61	20	41	66	16	38	56	19	49	61	34	73	83
Rouhani II(1970) <sup>4</sup>	5	17	29	10	29	45	23	42	58	19	37	49	11	25	53	7	36	56
Bonnecaze et al(1971)	4	21	38	11	25	50	6	28	58	11	28	53	9	26	53	21	43	66
Dix(1971)	26	38	44	35	50	63	34	56	63	21	44	51	30	52	56	44	63	72
Beggs(1972)	32	47	54	34	51	59	33	53	62	26	47	52	46	59	68	54	68	74
Chisholm(1973)	23	43	52	26	46	60	32	48	64	16	37	51	31	60	69	41	71	78
Loscher & Reinhardt(1973)	30	41	42	36	48	53	41	53	56	30	38	51	44	57	60	39	55	77
Mattar & Gregory(1974)	0	0	0	3	4	8	0	1	3	2	4	7	0	0	0	0	4	10
Moussali <sup>5</sup>	21	34	41	31	47	57	22	46	55	14	33	40	18	39	60	19	37	61
Greskovich & Cooper(1975)	23	43	51	40	56	64	29	53	67	30	49	57	27	53	63	27	53	72
Madsen(1975)	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0
Mukherjee(1979)	24	42	51	35	51	59	38	54	62	32	40	49	41	57	63	46	61	76
Gardner-1(1980)	11	20	24	16	31	41	15	29	38	11	27	33	7	19	32	10	16	34
Gardner-2(1980)	18	29	35	23	39	44	21	30	44	22	31	37	30	41	48	32	49	56
Sun et al(1980)	8	26	48	13	30	58	14	39	63	16	42	57	16	41	64	19	55	78
Jowitt(1981)	0	0	7	0	4	14	0	0	9	4	6	6	0	0	1	0	2	11
Chisholm(1983) - Armand(1946)	25	43	52	24	45	60	30	46	61	18	36	51	28	61	70	34	70	80
Spedding & Chen(1984)	20	24	36	21	33	44	14	22	31	15	22	32	17	26	43	21	39	54
Bestion(1985)	11	22	27	13	30	42	22	37	45	17	29	36	15	32	40	21	42	58
Tandon et al(1985)	10	23	32	12	24	34	8	17	25	9	20	34	14	24	42	19	35	50
Chen(1986)	32	47	50	35	54	56	40	51	55	29	41	47	43	49	53	44	56	62
Hamersma & Hart(1987)	10	19	27	14	25	30	4	13	21	7	11	19	9	21	30	14	27	36

Table 16. Number of data points correctly predicted for Mukherjee (1979) Inclined data (contd.)

Correlation / Inclination Angle	5 Degree data			20 Degree data			30 Degree data			50 Degree data			70 Degree data			80 Degree data		
	Total Points 57			Total Points 74			Total Points 74			Total Points 67			Total Points 80			Total Points 92		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%
Kawaji et al(1987)	10	21	28	13	25	36	10	15	19	9	19	28	10	19	38	14	34	46
Minami & Brill(1987)	33	44	52	41	53	62	48	57	62	27	46	51	45	54	62	43	67	75
El-Boher et al(1988)	21	36	41	30	53	59	32	50	57	14	28	43	18	39	58	17	54	66
Hart et al(1989)	18	27	31	20	28	34	13	17	27	12	22	29	16	30	39	31	38	47
Kokal & Stanislav(1989)	4	21	38	11	25	50	7	28	59	11	28	53	9	26	53	21	43	66
Spedding et al(1989)	20	28	37	20	31	36	14	22	29	13	22	31	16	25	43	21	45	51
Toshiba(1989)	23	45	47	26	50	64	37	58	61	26	43	50	31	52	55	38	70	76
Huq & Loth(1992)	23	40	47	27	44	53	23	43	53	20	33	41	30	54	63	33	53	80
Inoue et al(1993)	23	37	43	31	46	54	36	54	57	22	38	47	29	42	50	34	61	69
Czop et al(1994)	7	13	30	7	11	26	0	9	30	7	16	27	6	11	38	5	14	46
AbdulMajeed(1996)	21	33	36	32	43	46	26	36	41	19	25	37	28	40	46	26	40	49
Maier and Coddington(1997)	28	35	39	31	40	47	31	35	38	15	24	30	30	37	43	37	51	57
Petalaz & Aziz(1997)	2	5	8	2	5	7	2	6	10	2	4	7	7	9	11	4	8	10
Gomez et al(2000)	1	1	1	0	1	2	2	2	4	0	2	5	0	0	1	2	4	6
Zhao et al(2000)	0	1	4	2	3	6	0	1	3	0	0	0	0	0	0	0	1	3
Graham et al(2001)	19	31	36	24	39	49	22	35	50	19	34	42	18	43	51	37	56	70

correlation as reported by Leung (2005), <sup>2,3</sup> correlations as reported by Kataoka and Ishii (1987), <sup>3</sup> correlation as reported by Isbin and Biddle (1979), <sup>4</sup> Rouhani and Axelsson(1970) correlations I and II.

Table 17. Percentage of data points correctly predicted for Mukherjee (1979) Inclined data

Correlation / Inclination Angle	5 Degree data			20 Degree data			30 Degree data			50 Degree data			70 Degree data			80 Degree data		
	Total Points 57			Total Points 74			Total Points 74			Total Points 67			Total Points 80			Total Points 92		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%
Homogeneous	28.1	56.1	64.9	33.8	56.8	67.6	21.6	50.0	67.6	14.9	40.3	55.2	26.3	46.3	65.0	19.6	37.0	54.3
Armand(1946)	28.1	54.4	91.2	20.3	40.5	81.1	27.0	45.9	90.5	19.4	53.7	74.6	20.0	50.0	86.3	23.9	53.3	78.3
Armand - Massina <sup>1</sup>	40.4	73.7	94.7	27.0	60.8	87.8	37.8	63.5	91.9	25.4	64.2	76.1	33.8	73.8	87.5	42.4	78.3	85.9
Lockhart & Martinelli(1949)	38.6	57.9	75.4	31.1	54.1	63.5	23.0	41.9	66.2	23.9	47.8	62.7	35.0	70.0	83.8	37.0	68.5	84.8
Sterman(1956)	10.5	35.1	50.9	13.5	35.1	47.3	10.8	28.4	41.9	9.0	17.9	32.8	11.3	25.0	37.5	16.3	39.1	45.7
Filimonov et al(1957)	54.4	80.7	82.5	50.0	78.4	86.5	60.8	78.4	81.1	46.3	65.7	70.1	52.5	67.5	68.8	54.3	69.6	77.2
Chisholm & Laird(1958)	10.5	31.6	47.4	13.5	32.4	40.5	5.4	17.6	27.0	7.5	14.9	28.4	7.5	23.8	36.3	14.1	27.2	38.0
Flanigan(1958)	12.3	28.1	40.4	16.2	35.1	50.0	20.3	43.2	55.4	4.5	26.9	44.8	5.0	15.0	31.3	2.2	16.3	29.3
Dimentiev et al <sup>2</sup>	14.0	33.3	50.9	14.9	35.1	50.0	16.2	33.8	45.9	16.4	32.8	49.3	20.0	46.3	57.5	16.3	38.0	45.7
Hoogendroon(1959)	10.5	31.6	43.9	8.1	36.5	55.4	6.8	39.2	62.2	7.5	28.4	46.3	5.0	13.8	40.0	15.2	31.5	45.7
Bankoff(1960)	0.0	3.5	21.1	4.1	13.5	27.0	4.1	6.8	14.9	6.0	14.9	22.4	6.3	12.5	23.8	4.3	13.0	21.7
Fauske(1961)	17.5	22.8	29.8	13.5	25.7	29.7	10.8	13.5	16.2	16.4	23.9	29.9	13.8	21.3	25.0	23.9	30.4	33.7
Wilson et al(1961,1965)	12.3	26.3	36.8	13.5	33.8	43.2	14.9	36.5	52.7	14.9	32.8	40.3	12.5	20.0	31.3	13.0	19.6	21.7
Hughmark(1962)	29.8	68.4	93.0	28.4	60.8	81.1	31.1	62.2	89.2	28.4	58.2	80.6	23.8	63.8	77.5	34.8	75.0	88.0
Nicklin et al(1962)	7.0	36.8	66.7	14.9	33.8	67.6	8.1	37.8	78.4	16.4	41.8	79.1	11.3	32.5	66.3	22.8	46.7	71.7
Nishino & Yamazaki(1963)	36.8	47.4	68.4	25.7	43.2	54.1	20.3	32.4	43.2	22.4	38.8	47.8	17.5	41.3	56.3	23.9	44.6	60.9
Fujie(1964)	38.6	70.2	82.5	40.5	67.6	82.4	40.5	68.9	91.9	26.9	47.8	67.2	20.0	65.0	82.5	25.0	56.5	77.2
Kowalczewski(1964)	40.4	63.2	73.7	36.5	48.6	54.1	23.0	41.9	48.6	22.4	44.8	50.7	35.0	51.3	53.8	30.4	54.3	60.9
Thom(1964)	36.8	59.6	63.2	33.8	47.3	52.7	23.0	37.8	43.2	25.4	43.3	49.3	33.8	47.5	51.3	33.7	43.5	52.2
Zivi(1964)	36.8	52.6	61.4	31.1	40.5	43.2	18.9	23.0	29.7	17.9	31.3	43.3	26.3	40.0	47.5	31.5	39.1	53.3
Hughmark(1965)	28.1	42.1	77.2	16.2	37.8	68.9	21.6	43.2	87.8	17.9	40.3	68.7	18.8	45.0	76.3	23.9	50.0	71.7
Neal & Bankoff(1965)	19.3	24.6	28.1	13.5	21.6	29.7	9.5	12.2	14.9	6.0	7.5	14.9	15.0	30.0	31.3	5.4	10.9	13.0
Turner & Wallis(1965)	1.8	15.8	21.1	8.1	16.2	21.6	4.1	9.5	10.8	1.5	11.9	19.4	2.5	7.5	17.5	6.5	16.3	27.2
Baroczy(1966)	35.1	47.4	64.9	27.0	37.8	50.0	18.9	27.0	39.2	19.4	32.8	46.3	21.3	37.5	50.0	21.7	40.2	54.3
Guzhov et al(1967)	0.0	0.0	0.0	0.0	0.0	1.4	0.0	0.0	0.0	0.0	0.0	6.0	0.0	0.0	1.3	0.0	0.0	0.0
Kutucuglu <sup>3</sup>	28.1	49.1	59.6	24.3	39.2	54.1	18.9	29.7	33.8	19.4	34.3	47.8	37.5	50.0	56.3	28.3	39.1	50.0



Table 17. Percentage of data points correctly predicted for Mukherjee (1979) Inclined data (contd.)

Correlation / Inclination Angle	5 Degree data			20 Degree data			30 Degree data			50 Degree data			70 Degree data			80 Degree data		
	Total Points 57			Total Points 74			Total Points 74			Total Points 67			Total Points 80			Total Points 92		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%
Smith(1969)	42.1	73.7	84.2	39.2	59.5	73.0	33.8	66.2	77.0	25.4	53.7	73.1	43.8	72.5	85.0	43.5	73.9	85.9
Wallis(1969)	38.6	50.9	75.4	32.4	48.6	64.9	24.3	41.9	70.3	23.9	46.3	64.2	31.3	66.3	80.0	27.2	56.5	76.1
Gregory & Scott(1969)	28.1	57.9	96.5	17.6	40.5	90.5	28.4	51.4	89.2	19.4	53.7	76.1	20.0	51.3	87.5	28.3	56.5	87.0
Premoli et al(1970)	54.4	73.7	82.5	56.8	68.9	81.1	58.1	75.7	82.4	29.9	62.7	80.6	52.5	71.3	77.5	46.7	63.0	78.3
Rouhani I(1970)4	22.8	61.4	91.2	25.7	54.1	82.4	27.0	55.4	89.2	23.9	56.7	83.6	23.8	61.3	76.3	37.0	79.3	90.2
Rouhani II(1970)4	8.8	29.8	50.9	13.5	39.2	60.8	31.1	56.8	78.4	28.4	55.2	73.1	13.8	31.3	66.3	7.6	39.1	60.9
Bonnecaze et al(1971)	7.0	36.8	66.7	14.9	33.8	67.6	8.1	37.8	78.4	16.4	41.8	79.1	11.3	32.5	66.3	22.8	46.7	71.7
Dix(1971)	45.6	66.7	77.2	47.3	67.6	85.1	45.9	75.7	85.1	31.3	65.7	76.1	37.5	65.0	70.0	47.8	68.5	78.3
Beggs(1972)	56.1	82.5	94.7	45.9	68.9	79.7	44.6	71.6	83.8	38.8	70.1	77.6	57.5	73.8	85.0	58.7	73.9	80.4
Chisholm(1973)	40.4	75.4	91.2	35.1	62.2	81.1	43.2	64.9	86.5	23.9	55.2	76.1	38.8	75.0	86.3	44.6	77.2	84.8
Loscher & Reinhardt(1973)	52.6	71.9	73.7	48.6	64.9	71.6	55.4	71.6	75.7	44.8	56.7	76.1	55.0	71.3	75.0	42.4	59.8	83.7
Mattar & Gregory(1974)	0.0	0.0	0.0	4.1	5.4	10.8	0.0	1.4	4.1	3.0	6.0	10.4	0.0	0.0	0.0	0.0	4.3	10.9
Moussali5	36.8	59.6	71.9	41.9	63.5	77.0	29.7	62.2	74.3	20.9	49.3	59.7	22.5	48.8	75.0	20.7	40.2	66.3
Greskovitch & Cooper(1975)	40.4	75.4	89.5	54.1	75.7	86.5	39.2	71.6	90.5	44.8	73.1	85.1	33.8	66.3	78.8	29.3	57.6	78.3
Madson(1975)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.0	0.0	0.0	0.0	0.0	0.0	0.0
Mukherjee(1979)	42.1	73.7	89.5	47.3	68.9	79.7	51.4	73.0	83.8	47.8	59.7	73.1	51.3	71.3	78.8	50.0	66.3	82.6
Gardner-1(1980)	19.3	35.1	42.1	21.6	41.9	55.4	20.3	39.2	51.4	16.4	40.3	49.3	8.8	23.8	40.0	10.9	17.4	37.0
Gardner-2(1980)	31.6	50.9	61.4	31.1	52.7	59.5	28.4	40.5	59.5	32.8	46.3	55.2	37.5	51.3	60.0	34.8	53.3	60.9
Sun et al(1980)	14.0	45.6	84.2	17.6	40.5	78.4	18.9	52.7	85.1	23.9	62.7	85.1	20.0	51.3	80.0	20.7	59.8	84.8
Jowitt(1981)	0.0	0.0	12.3	0.0	5.4	18.9	0.0	0.0	12.2	6.0	9.0	9.0	0.0	0.0	1.3	0.0	2.2	12.0
Chisholm(1983) - Armand(1946)	43.9	75.4	91.2	32.4	60.8	81.1	40.5	62.2	82.4	26.9	53.7	76.1	35.0	76.3	87.5	37.0	76.1	87.0
Spedding & Chen(1984)	35.1	42.1	63.2	28.4	44.6	59.5	18.9	29.7	41.9	22.4	32.8	47.8	21.3	32.5	53.8	22.8	42.4	58.7
Bestion(1985)	19.3	38.6	47.4	17.6	40.5	56.8	29.7	50.0	60.8	25.4	43.3	53.7	18.8	40.0	50.0	22.8	45.7	63.0
Tandon et al(1985)	17.5	40.4	56.1	16.2	32.4	45.9	10.8	23.0	33.8	13.4	29.9	50.7	17.5	30.0	52.5	20.7	38.0	54.3
Chen(1986)	56.1	82.5	87.7	47.3	73.0	75.7	54.1	68.9	74.3	43.3	61.2	70.1	53.8	61.3	66.3	47.8	60.9	67.4
Hamersma & Hart(1987)	17.5	33.3	47.4	18.9	33.8	40.5	5.4	17.6	28.4	10.4	16.4	28.4	11.3	26.3	37.5	15.2	29.3	39.1

Table 17. Percentage of data points correctly predicted for Mukherjee (1979) Inclined data (contd.)

Correlation / Inclination Angle	5 Degree data			20 Degree data			30 Degree data			50 Degree data			70 Degree data			80 Degree data		
	Total Points 57			Total Points 74			Total Points 74			Total Points 67			Total Points 80			Total Points 92		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%
Kawaji et al(1987)	17.5	36.8	49.1	17.6	33.8	48.6	13.5	20.3	25.7	13.4	28.4	41.8	12.5	23.8	47.5	15.2	37.0	50.0
Minami & Brill(1987)	57.9	77.2	91.2	55.4	71.6	83.8	64.9	77.0	83.8	40.3	68.7	76.1	56.3	67.5	77.5	46.7	72.8	81.5
El-Boher et al(1988)	36.8	63.2	71.9	40.5	71.6	79.7	43.2	67.6	77.0	20.9	41.8	64.2	22.5	48.8	72.5	18.5	58.7	71.7
Hart et al(1989)	31.6	47.4	54.4	27.0	37.8	45.9	17.6	23.0	36.5	17.9	32.8	43.3	20.0	37.5	48.8	33.7	41.3	51.1
Kokal & Stanislav(1989)	7.0	36.8	66.7	14.9	33.8	67.6	9.5	37.8	79.7	16.4	41.8	79.1	11.3	32.5	66.3	22.8	46.7	71.7
Spedding et al(1989)	35.1	49.1	64.9	27.0	41.9	48.6	18.9	29.7	39.2	19.4	32.8	46.3	20.0	31.3	53.8	22.8	48.9	55.4
Toshiba(1989)	40.4	78.9	82.5	35.1	67.6	86.5	50.0	78.4	82.4	38.8	64.2	74.6	38.8	65.0	68.8	41.3	76.1	82.6
Huq & Loth(1992)	40.4	70.2	82.5	36.5	59.5	71.6	31.1	58.1	71.6	29.9	49.3	61.2	37.5	67.5	78.8	35.9	57.6	87.0
Inoue et al(1993)	40.4	64.9	75.4	41.9	62.2	73.0	48.6	73.0	77.0	32.8	56.7	70.1	36.3	52.5	62.5	37.0	66.3	75.0
Czop et al(1994)	12.3	22.8	52.6	9.5	14.9	35.1	0.0	12.2	40.5	10.4	23.9	40.3	7.5	13.8	47.5	5.4	15.2	50.0
AbdulMajeed(1996)	36.8	57.9	63.2	43.2	58.1	62.2	35.1	48.6	55.4	28.4	37.3	55.2	35.0	50.0	57.5	28.3	43.5	53.3
Maier and Coddington(1997)	49.1	61.4	68.4	41.9	54.1	63.5	41.9	47.3	51.4	22.4	35.8	44.8	37.5	46.3	53.8	40.2	55.4	62.0
Petalaz & Aziz(1997)	3.5	8.8	14.0	2.7	6.8	9.5	2.7	8.1	13.5	3.0	6.0	10.4	8.8	11.3	13.8	4.3	8.7	10.9
Gomez et al(2000)	1.8	1.8	1.8	0.0	1.4	2.7	2.7	2.7	5.4	0.0	3.0	7.5	0.0	0.0	1.3	2.2	4.3	6.5
Zhao et al(2000)	0.0	1.8	7.0	2.7	4.1	8.1	0.0	1.4	4.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.1	3.3
Graham et al(2001)	33.3	54.4	63.2	32.4	52.7	66.2	29.7	47.3	67.6	28.4	50.7	62.7	22.5	53.8	63.8	40.2	60.9	76.1

<sup>1</sup> correlation as reported by Leung (2005), <sup>2,5</sup> correlations as reported by Kataoka and Ishii (1987), <sup>3</sup> correlation as reported by Isbin and Biddle (1979), <sup>4</sup> Rouhani and Axelsson(1970) correlations I and II.

Table 18. Total Number and percentage of data points correctly predicted for the inclined experimental data of Mukherjee (1979)

Correlation / Inclination Angle	Total data points correctly predicted for the whole database			Percentage of data points correctly predicted for the whole database		
	Total Points 444			Total Points 444		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%
Homogeneous	106	209	276	23.9	47.1	62.2
Armand(1946)	102	220	370	23.0	49.5	83.3
Armand - Massina <sup>1</sup>	154	308	387	34.7	69.4	87.2
Lockhart & Martinelli(1949)	140	255	326	31.5	57.4	73.4
Sterman(1956)	54	135	189	12.2	30.4	42.6
Filimonov et al(1957)	236	324	344	53.2	73.0	77.5
Chisholm & Laird(1958)	44	109	160	9.9	24.5	36.0
Flanigan(1958)	43	119	183	9.7	26.8	41.2
Dimentiev et al <sup>2</sup>	73	164	221	16.4	36.9	49.8
Hoogendroon(1959)	40	133	217	9.0	30.0	48.9
Bankoff(1960)	19	49	97	4.3	11.0	21.8
Fauske(1961)	72	103	122	16.2	23.2	27.5
Wilson et al(1961,1965)	60	123	164	13.5	27.7	36.9
Hughmark(1962)	131	289	376	29.5	65.1	84.7
Nicklin et al(1962)	62	171	318	14.0	38.5	71.6
Nishino & Yamazaki(1963)	106	183	244	23.9	41.2	55.0
Fujie(1964)	139	277	358	31.3	62.4	80.6
Kowalczewski(1964)	138	224	251	31.1	50.5	56.5
Thom(1964)	138	204	229	31.1	45.9	51.6
Zivi(1964)	120	166	205	27.0	37.4	46.2
Hughmark(1965)	93	193	333	20.9	43.5	75.0
Neal & Bankoff(1965)	49	78	96	11.0	17.6	21.6
Turner & Wallis(1965)	19	57	88	4.3	12.8	19.8
Baroczy(1966)	104	164	224	23.4	36.9	50.5
Guzhov et al(1967)	0	0	6	0.0	0.0	1.4
Kutucuglu <sup>3</sup>	117	178	222	26.4	40.1	50.0
Smith(1969)	170	297	355	38.3	66.9	80.0
Wallis(1969)	130	232	320	29.3	52.3	72.1
Gregory & Scott(1969)	105	230	389	23.6	51.8	87.6
Premoli et al(1970)	221	306	356	49.8	68.9	80.2
Rouhani I(1970) <sup>4</sup>	121	276	379	27.3	62.2	85.4
Rouhani II(1970) <sup>4</sup>	75	186	290	16.9	41.9	65.3
Bonnecaze et al(1971)	62	171	318	14.0	38.5	71.6
Dix(1971)	190	303	349	42.8	68.2	78.6
Beggs(1972)	225	325	369	50.7	73.2	83.1
Chisholm(1973)	169	305	374	38.1	68.7	84.2
Loscher & Reinhardt(1973)	220	292	339	49.5	65.8	76.4
Mattar & Gregory(1974)	5	13	28	1.1	2.9	6.3
Moussali <sup>5</sup>	125	236	314	28.2	53.2	70.7
Greskovich & Cooper(1975)	176	307	374	39.6	69.1	84.2
Madsen(1975)	0	0	2	0.0	0.0	0.5
Mukherjee(1979)	216	305	360	48.6	68.7	81.1

Table 18. Total Number and percentage of data points correctly predicted for the inclined experimental data of Mukherjee (1979)(contd.)

Correlation / Inclination Angle	Total data points correctly predicted for the whole database			Percentage of data points correctly predicted for the whole database		
	Total Points 444			Total Points 444		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%
Gardner-1(1980)	70	142	202	15.8	32.0	45.5
Gardner-2(1980)	146	219	264	32.9	49.3	59.5
Sun et al(1980)	86	233	368	19.4	52.5	82.9
Jowitt(1981)	4	12	48	0.9	2.7	10.8
Chisholm(1983) - Armand(1946)	159	301	374	35.8	67.8	84.2
Spedding & Chen(1984)	108	166	240	24.3	37.4	54.1
Bestion(1985)	99	192	248	22.3	43.2	55.9
Tandon et al(1985)	72	143	217	16.2	32.2	48.9
Chen(1986)	223	298	323	50.2	67.1	72.7
Hamersma & Hart(1987)	58	116	163	13.1	26.1	36.7
Kawaji et al(1987)	66	133	195	14.9	30.0	43.9
Minami & Brill(1987)	237	321	364	53.4	72.3	82.0
El-Boher et al(1988)	132	260	324	29.7	58.6	73.0
Hart et al(1989)	110	162	207	24.8	36.5	46.6
Kokal & Stanislav(1989)	63	171	319	14.2	38.5	71.8
Spedding et al(1989)	104	173	227	23.4	39.0	51.1
Toshiba(1989)	181	318	353	40.8	71.6	79.5
Huq & Loth(1992)	156	267	337	35.1	60.1	75.9
Inoue et al(1993)	175	278	320	39.4	62.6	72.1
Czop et al(1994)	32	74	197	7.2	16.7	44.4
AbdulMajeed(1996)	152	217	255	34.2	48.9	57.4
Maier and Coddington(1997)	172	222	254	38.7	50.0	57.2
Petalaz & Aziz(1997)	19	37	53	4.3	8.3	11.9
Gomez et al(2000)	5	10	19	1.1	2.3	4.3
Zhao et al(2000)	2	6	16	0.5	1.4	3.6
Graham et al(2001)	139	238	298	31.3	53.6	67.1

<sup>1</sup> correlation as reported by Leung (2005), <sup>2,5</sup> correlations as reported by Kataoka and Ishii (1987), <sup>3</sup> correlation as reported by Isbin and Biddle (1979), <sup>4</sup> Rouhani and Axelsson(1970) correlations I and II.

On the other hand, for the total inclined air-kerosene data of Mukherjee(1979), the best prediction was given by Gregory and Scott(1969), Armand Massina (Leung 2005), Rouhani I(1970) and Hughmark(1962) predicting 87.6,87.2,85.4 and 84.7% of the data within 15% error margin respectively. The excellent prediction by the simple drift flux correlation of Gregory and Scott (1969) that was developed for horizontal slug flow could only be explained by the fact that most of the data in the set considered is dominated by slug flow regime. However, the simple deduction that the

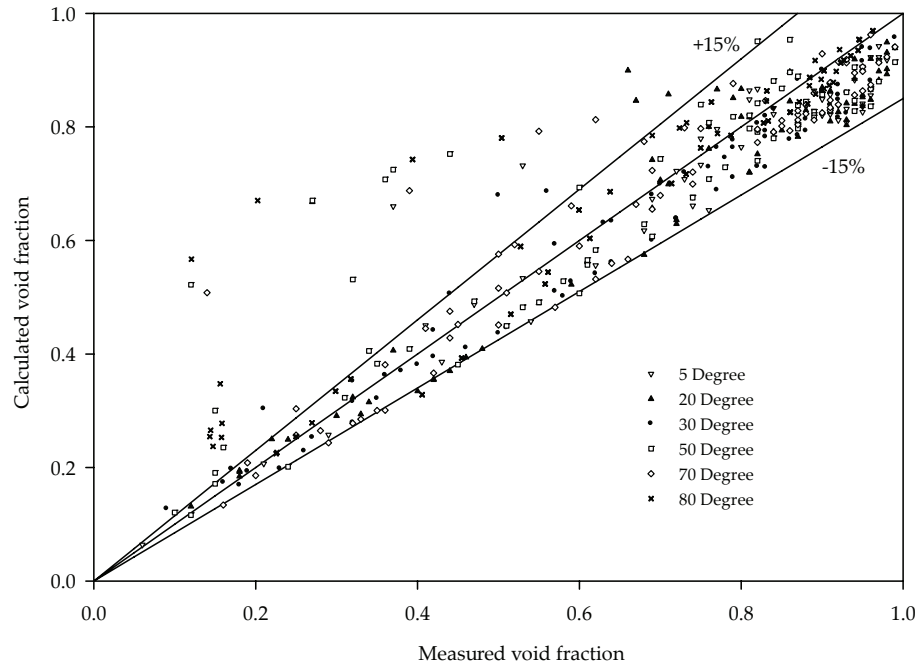


Figure 21. Comparison of Armand Massina (Leung 2005) correlation with measured inclined experimental data of Mukherjee (1979)

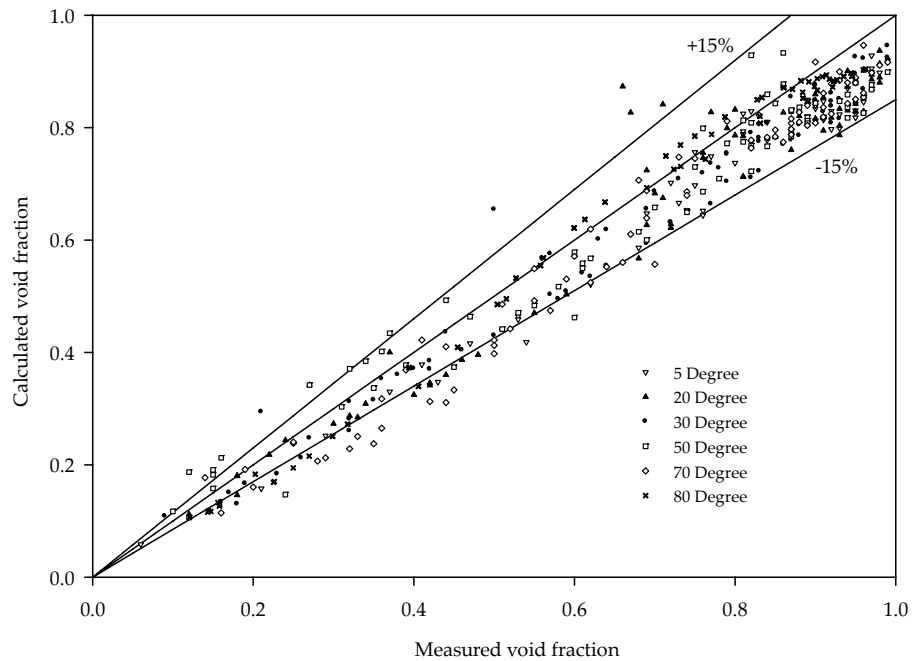


Figure 22. Comparison of Rouhani I (1970) correlation with measured inclined experimental data of Mukherjee (1979)

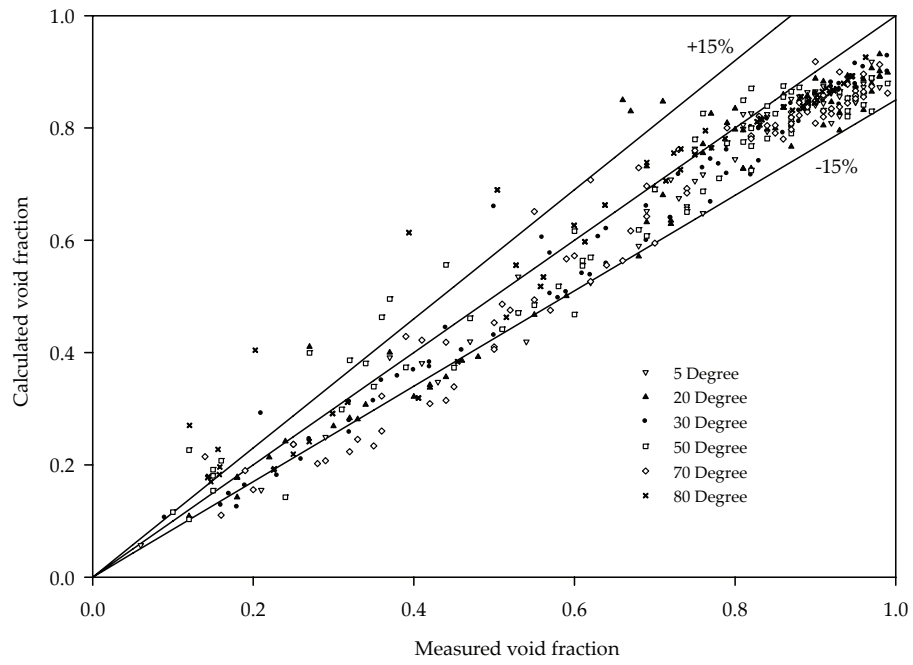


Figure 23. Comparison of Hughmark (1962) correlation with measured inclined experimental data of Mukherjee (1979)

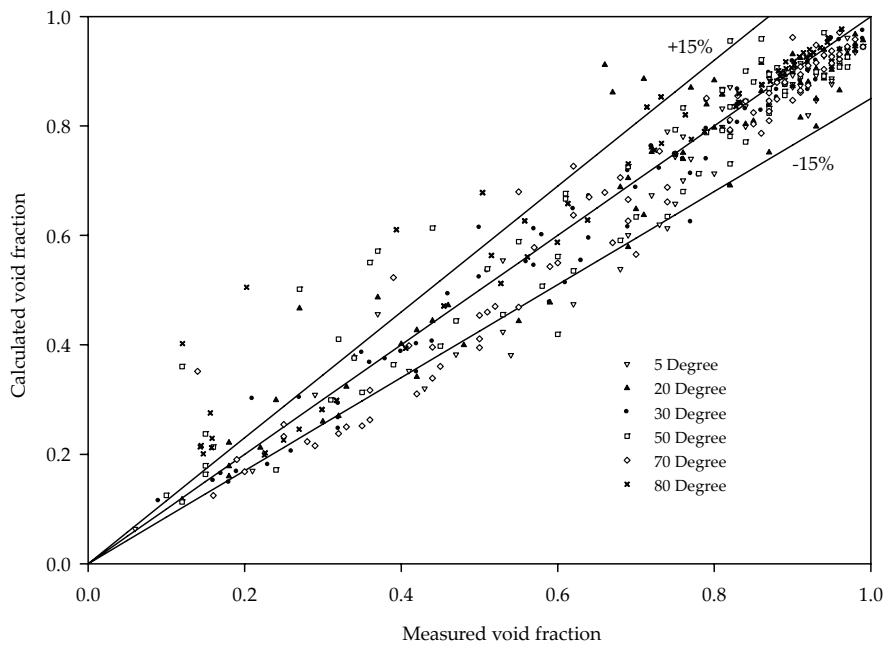


Figure 24. Comparison of Premoli et al. (1970) correlation with measured inclined experimental data of Mukherjee (1979)

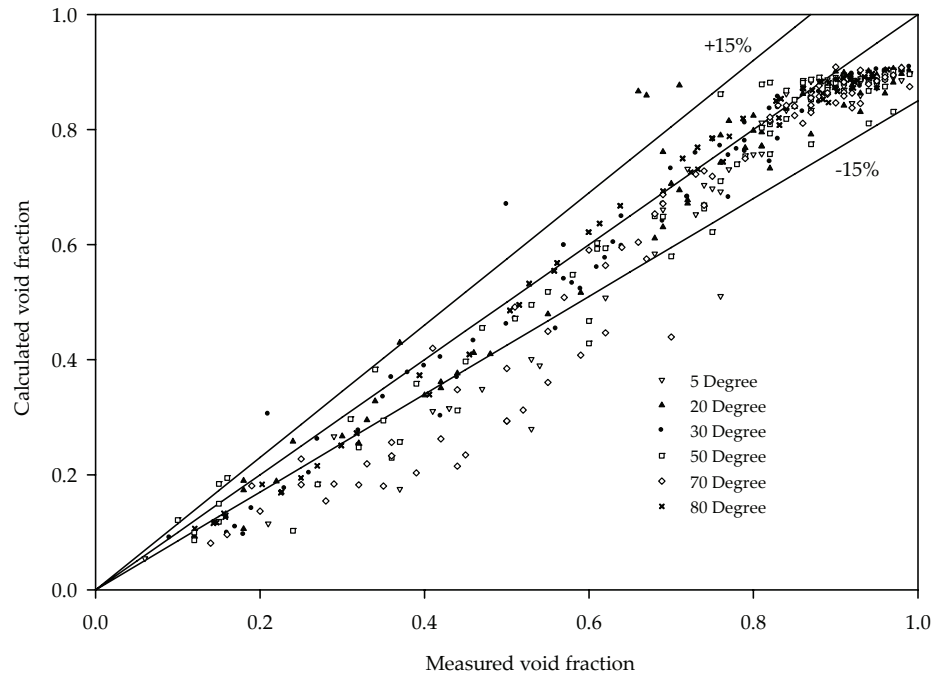


Figure 25. Comparison of Toshiba (1989) correlation with measured inclined experimental data of Mukherjee (1979)

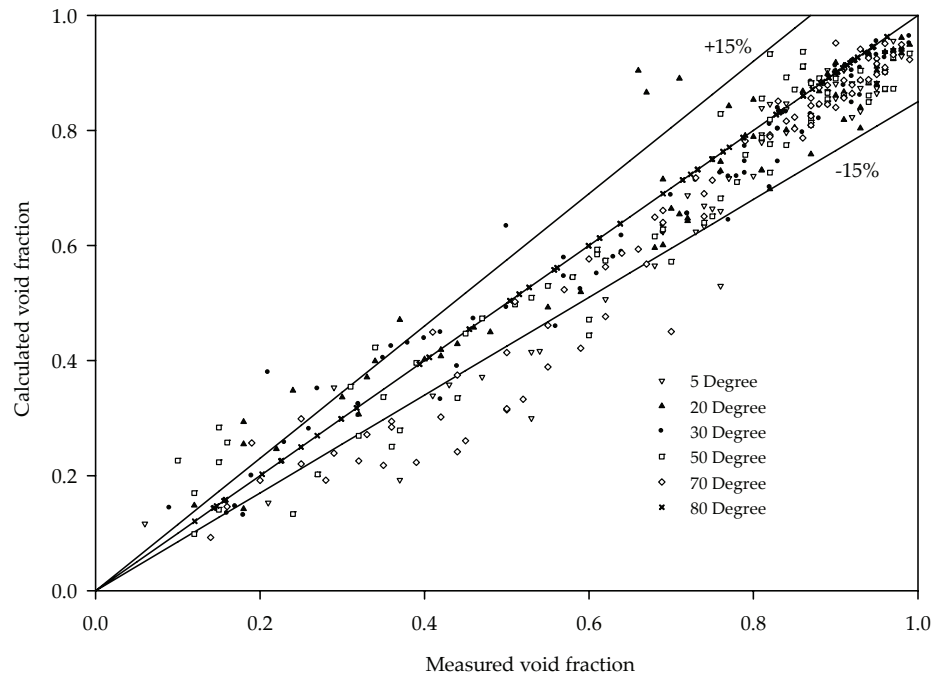


Figure 26. Comparison of Dix (1971) correlation with measured inclined experimental data of Mukherjee (1979)

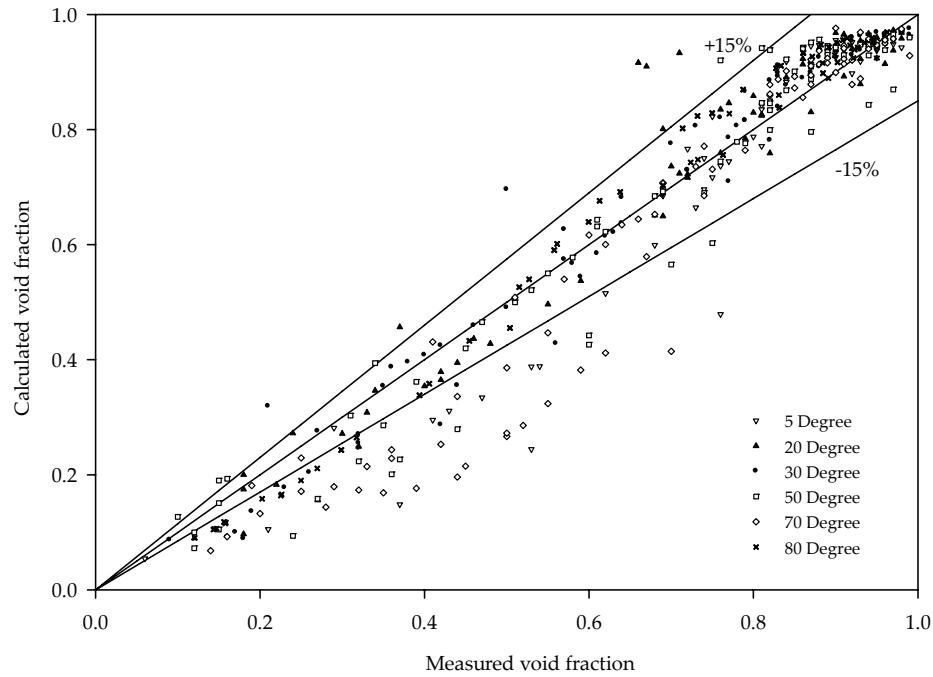


Figure 27. Comparison of Filimonov et al. (1957) correlation with measured inclined experimental data of Mukherjee (1979)

drift velocity between the phases is zero which could be taken from the drift flux correlation expression would be quite erroneous. We have presented the horizontal and inclined comparisons so far and what remains to be discussed is the vertical flow comparison which will follow next.

### 4.3 Vertical flow comparison

The presentation of the comparison in this section would be very similar to that of the horizontal comparison. A total of 403 data points are included for these comparisons from four sources namely, Beggs (1972), Spedding and Nguyen (1976), Mukherjee (1979) and Sujumnong (1997).

Over half of the data points in this data set are from Spedding and Nguyen (1976) which would influence most of the final conclusions and recommendations.



Most of the general observations and comments on the horizontal and inclined comparison would also hold for this section.

The total number of data points correctly predicted within each error index for each data set is given in Table 20. The respective percentage figures are presented in Table 21.

The correlation by Beggs (1972) and Minami and Brill (1987) were the only ones that made an acceptably close prediction within the 5% index for the data of Beggs (1972). None of the correlations were able to predict a sufficient percentage of the data points in any of the data sets within this range. The performance shows improvement for the wider error bands for all the data sets except the data of Spedding and Nguyen (1976).

This has been the case for all the previous comparisons. The only new correlation that came into the picture is that of El-Boher et al. (1988) which gave an 83.5% prediction of the Spedding and Nguyen (1976) data within the 15% error index even though it was not able to satisfactorily predict the data of Sujumnong (1997).

Considering their general performance across the three other data sets, the correlation by Armand – Massina (Leung 2005), Filimonov et al. (1957), Hughmark (1962), Smith (1969), Gregory & Scott (1969), Premoli et al. (1970), Rouhani I(1970), Chisholm (1973), Chisholm (1983) Armand (1946), Toshiba (1989) and Huq and Loth (1992) have acceptable prediction and are worth consideration.

Consolidating our observation of the separate data sets into one, we have presented the predictive capability of the correlations for the whole vertical data base in Table 22. The drift flux correlations by Toshiba (1989), Rouhani I (1970) and Filimonov et al. (1957) once again have shown an excellent performance in predicting the vertical data base within the 15% index. Six other correlations have given a fairly good result

Table 19. Total Number and percentage of data points correctly predicted for the whole inclined data base

Correlation / Data Source	Total data points correctly predicted for the whole database			Percentage of data points correctly predicted for the whole database		
	Total Points 1542			Total Points 1542		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%
Homogeneous	345	544	698	22.4	35.3	45.3
Armand(1946)	357	646	999	23.2	41.9	64.8
Armand - Massina <sup>1</sup>	609	1008	1163	39.5	65.4	75.4
Lockhart & Martinelli(1949)	680	959	1119	44.1	62.2	72.6
Sterman(1956)	311	488	614	20.2	31.6	39.8
Filimonov et al(1957)	728	1049	1239	47.2	68.0	80.4
Chisholm & Laird(1958)	273	411	525	17.7	26.7	34.0
Flanigan(1958)	315	557	715	20.4	36.1	46.4
Dimentiev et al <sup>2</sup>	367	601	777	23.8	39.0	50.4
Hoogendroon(1959)	329	600	794	21.3	38.9	51.5
Bankoff(1960)	115	258	478	7.5	16.7	31.0
Fauske(1961)	351	467	527	22.8	30.3	34.2
Wilson et al(1961,1965)	148	312	419	9.6	20.2	27.2
Hughmark(1962)	680	1025	1196	44.1	66.5	77.6
Nicklin et al(1962)	341	684	1040	22.1	44.4	67.4
Nishino & Yamazaki(1963)	633	863	997	41.1	56.0	64.7
Fujie(1964)	428	712	938	27.8	46.2	60.8
Kowalczewski(1964)	607	821	895	39.4	53.2	58.0
Thom(1964)	479	735	868	31.1	47.7	56.3
Zivi(1964)	533	672	775	34.6	43.6	50.3
Hughmark(1965)	340	607	925	22.0	39.4	60.0
Neal & Bankoff(1965)	163	283	384	10.6	18.4	24.9
Turner & Wallis(1965)	212	334	418	13.7	21.7	27.1
Baroczy(1966)	574	777	911	37.2	50.4	59.1
Guzhov et al(1967)	5	56	156	0.3	3.6	10.1
Kutucuglu <sup>3</sup>	530	703	800	34.4	45.6	51.9
Smith(1969)	684	1003	1132	44.4	65.0	73.4
Wallis(1969)	673	933	1104	43.6	60.5	71.6
Gregory & Scott(1969)	359	661	1074	23.3	42.9	69.6
Premoli et al(1970)	834	1090	1211	54.1	70.7	78.5
Rouhani I(1970) <sup>4</sup>	604	1053	1243	39.2	68.3	80.6
Rouhani II(1970) <sup>4</sup>	367	703	1006	23.8	45.6	65.2
Bonnecaze et al(1971)	342	684	1040	22.2	44.4	67.4
Dix(1971)	868	1167	1272	56.3	75.7	82.5
Beggs(1972)	637	937	1096	41.3	60.8	71.1
Chisholm(1973)	696	998	1149	45.1	64.7	74.5
Loscher & Reinhardt(1973)	633	761	865	41.1	49.4	56.1
Mattar & Gregory(1974)	27	102	279	1.8	6.6	18.1
Moussali <sup>5</sup>	384	604	778	24.9	39.2	50.5
Greskovich & Cooper(1975)	496	773	1002	32.2	50.1	65.0
Madsen(1975)	54	87	105	3.5	5.6	6.8
Mukherjee(1979)	639	941	1146	41.4	61.0	74.3

Table 19. Total Number and percentage of data points correctly predicted for the whole inclined data base(contd.)

Correlation / Data Source	Total data points correctly predicted for the whole database			Percentage of data points correctly predicted for the whole database		
	Total Points 1542			Total Points 1542		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%
Gardner-1(1980)	379	613	781	24.6	39.8	50.6
Gardner-2(1980)	469	727	873	30.4	47.1	56.6
Sun et al(1980)	338	715	1043	21.9	46.4	67.6
Jowitt(1981)	153	308	614	9.9	20.0	39.8
Chisholm(1983) - Armand(1946)	702	1003	1152	45.5	65.0	74.7
Spedding & Chen(1984)	606	809	973	39.3	52.5	63.1
Bestion(1985)	378	644	800	24.5	41.8	51.9
Tandon et al(1985)	510	696	851	33.1	45.1	55.2
Chen(1986)	683	920	1035	44.3	59.7	67.1
Hamersma & Hart(1987)	290	428	532	18.8	27.8	34.5
Kawaji et al(1987)	397	645	795	25.7	41.8	51.6
Minami & Brill(1987)	650	933	1110	42.2	60.5	72.0
El-Boher et al(1988)	700	1034	1168	45.4	67.1	75.7
Hart et al(1989)	557	689	788	36.1	44.7	51.1
Kokal & Stanislav(1989)	340	680	1037	22.0	44.1	67.3
Spedding et al(1989)	551	727	833	35.7	47.1	54.0
Toshiba(1989)	595	1165	1313	38.6	75.6	85.1
Huq & Loth(1992)	692	988	1125	44.9	64.1	73.0
Inoue et al(1993)	585	909	1088	37.9	58.9	70.6
Czop et al(1994)	209	420	704	13.6	27.2	45.7
AbdulMajeed(1996)	461	755	895	29.9	49.0	58.0
Maier and Coddington(1997)	550	790	909	35.7	51.2	58.9
Petalaz & Aziz(1997)	51	114	170	3.3	7.4	11.0
Gomez et al(2000)	246	371	452	16.0	24.1	29.3
Zhao et al(2000)	160	229	285	10.4	14.9	18.5
Graham et al(2001)	654	919	1074	42.4	59.6	69.6

<sup>1</sup> correlation as reported by Leung (2005), <sup>2,5</sup> correlations as reported by Kataoka and Ishii (1987), <sup>3</sup> correlation as reported by Isbin and Biddle (1979), <sup>4</sup> Rouhani and Axelsson(1970) correlations I and II.

within this range. These are, Hughmark(1962), Nicklin et al.(1962), Premoli et al.(1970), Bonnacaze et al.(1971), Dix(1971) and Kokal and Stanislav(1989). The correlations of Hughmark (1962), Premoli et al. (1970) and Dix (1971) are worth a general recommendation while the other three being discarded as unsatisfactory as could be seen from their poor performance within the 10% error index.

We have again implemented our simple scatter check to see the consistency of the top performing six correlations presented in Figures 35 to 40. Toshiba (1989) has once again showed a very good consistency with only a few “stray” under predictions.

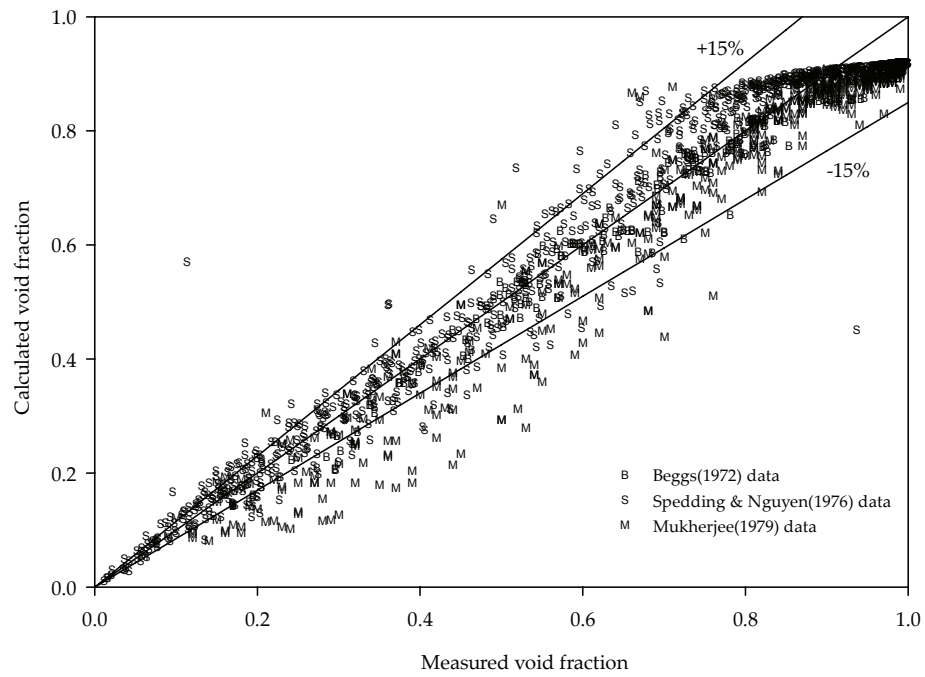


Figure 28. Comparison of Toshiba (1989) correlation with measured combined inclined experimental data

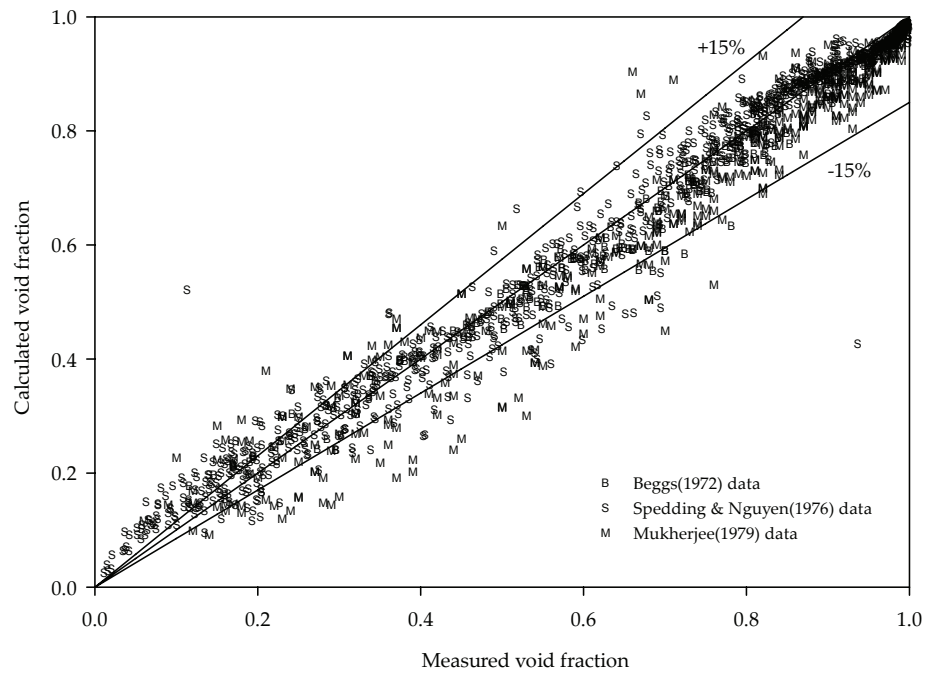


Figure 29. Comparison of Dix (1971) correlation with measured combined inclined experimental data

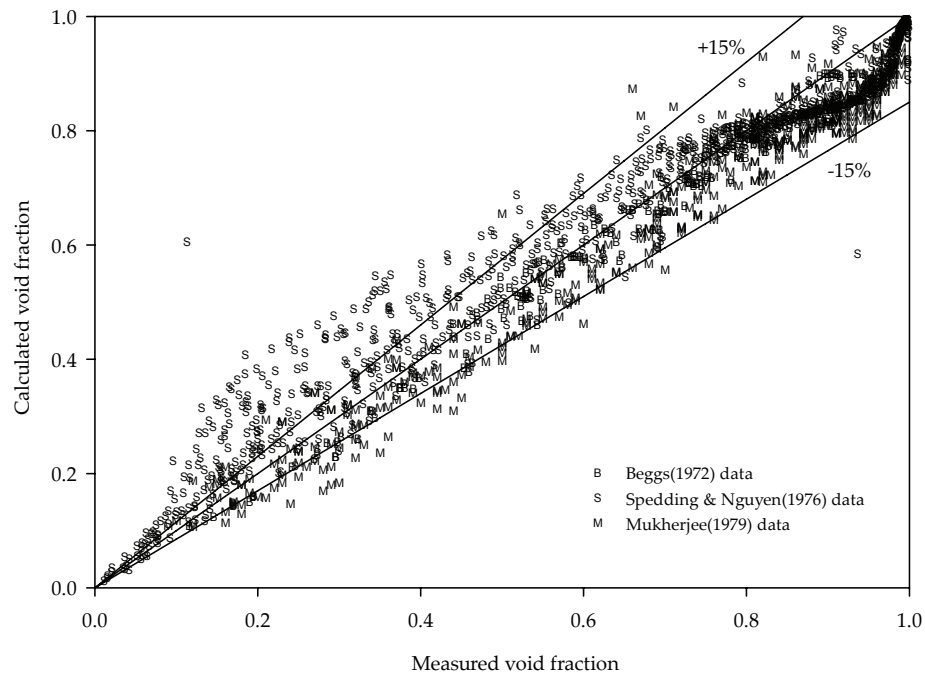


Figure 30. Comparison of Rouhani I (1970) correlation with measured combined inclined experimental data

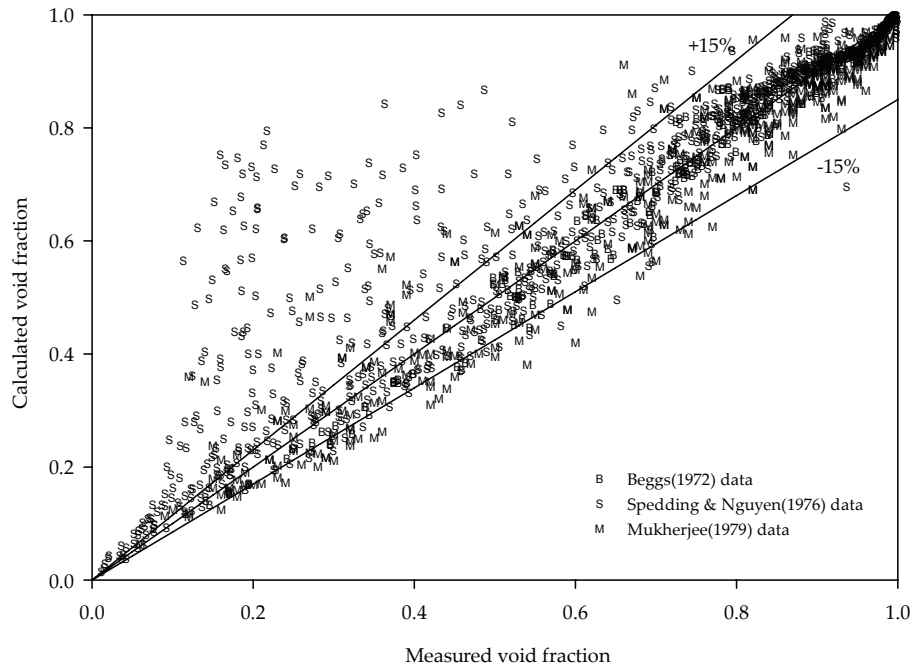


Figure 31. Comparison of Premoli et al. (1970) correlation with measured combined inclined experimental data

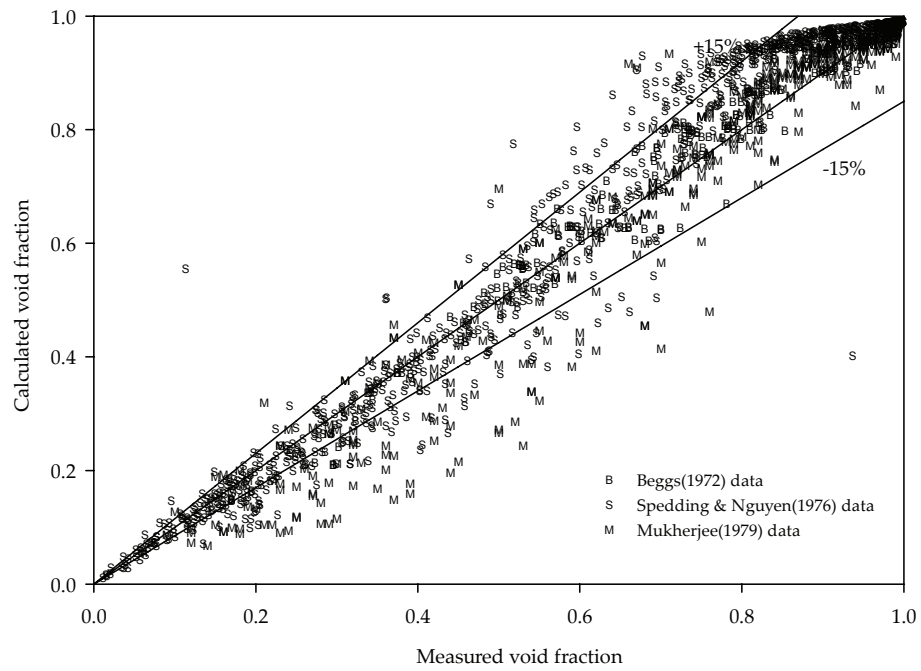


Figure 32. Comparison of Filimonov et al. (1957) correlation with measured combined inclined experimental data

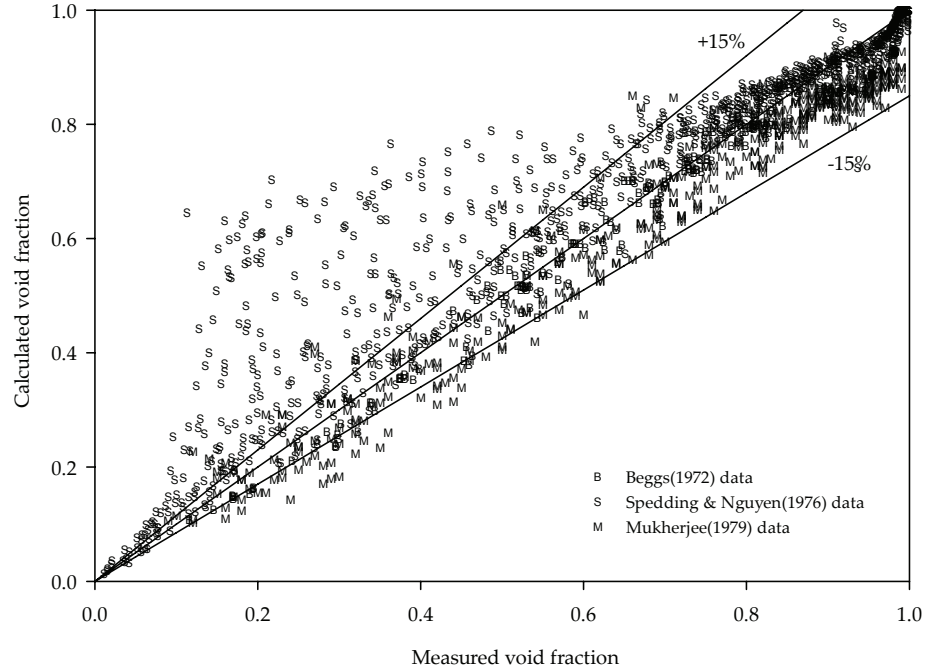


Figure 33. Comparison of Hughmark (1962) correlation with measured combined inclined experimental data

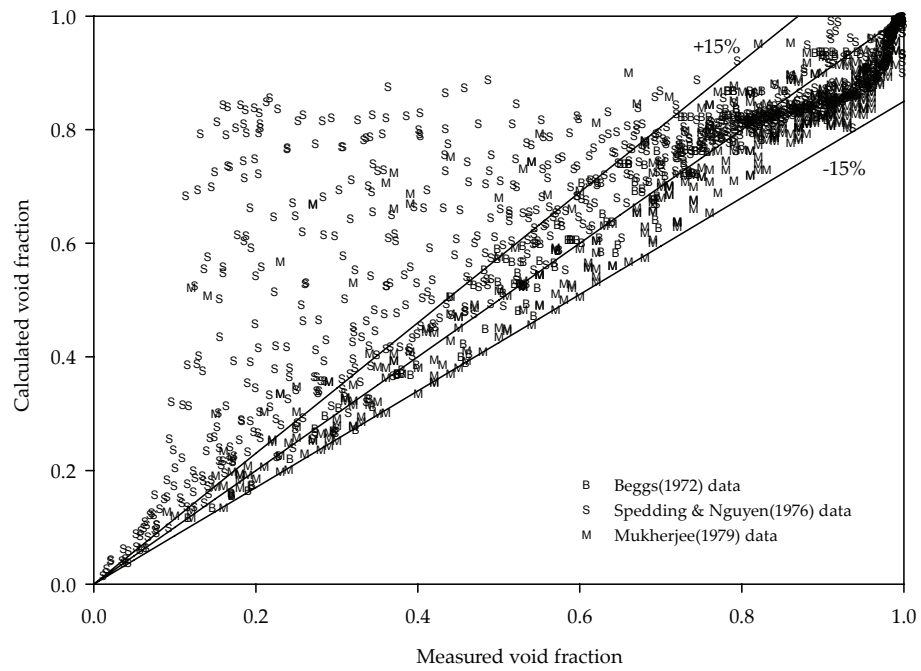


Figure 34. Comparison of Armand Massina (Leung 2005) correlation with measured combined inclined experimental data

Table 20. Number of data points correctly predicted for vertical flow by data set

Correlation / Data Source	Beggs (1972) (Air -Water)			Spedding & Nguyen (1976) (Air -Water)			Mukherjee (1979) (Air -Kerosene)			Sujumnong (1997) (Air -Water)		
	Total Points 26			Total Points 224			Total Points 52			Total Points 101		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%
Homogeneous	5	9	14	34	55	79	13	27	33	24	40	59
Armand(1946)	12	18	23	47	84	135	16	25	46	35	60	85
Armand - Massina <sup>1</sup>	17	24	25	70	131	147	25	40	47	44	77	92
Lockhart & Martinelli(1949)	13	20	25	95	126	151	22	33	44	46	56	63
Sterman(1956)	6	10	11	47	73	90	6	22	26	15	26	33
Filimonov et al(1957)	19	24	24	77	136	180	33	42	45	38	69	89
Chisholm & Laird(1958)	4	6	7	33	53	68	4	19	26	13	21	26
Flanigan(1958)	2	5	9	56	81	101	1	9	19	26	35	40
Dimentiev et al <sup>2</sup>	7	9	13	45	79	99	11	31	33	21	30	37
Hoogendroon(1959)	0	4	9	63	102	117	4	9	25	44	64	75
Bankoff(1960)	0	2	5	32	54	81	0	0	12	2	7	31
Fauske(1961)	7	7	8	41	61	71	10	24	26	19	24	26
Wilson et al(1961,1965)	3	7	10	17	37	60	6	12	18	3	5	13
Hughmark(1962)	10	24	25	127	160	169	16	39	46	69	79	89
Nicklin et al(1962)	3	14	23	90	140	181	9	18	40	33	60	85
Nishino & Yamazaki(1963)	9	15	17	108	141	167	23	31	38	39	47	52
Fujie(1964)	7	15	20	39	70	111	16	31	38	32	58	79
Kowalczewski(1964)	10	13	15	99	118	137	22	34	37	37	49	52
Thom(1964)	9	13	16	62	110	127	19	28	35	20	37	47
Zivi(1964)	9	12	14	82	102	118	23	31	37	28	40	42
Hughmark(1965)	9	17	23	39	82	124	14	22	38	35	56	79
Neal & Bankoff(1965)	3	5	6	24	45	67	6	8	9	6	11	20
Turner & Wallis(1965)	3	5	7	24	39	60	5	6	13	7	13	22
Baroczy(1966)	9	14	15	96	124	150	25	31	37	37	48	49
Guzhov et al(1967)	0	0	0	0	0	21	0	0	0	0	0	4
Kutucuglu <sup>3</sup>	6	12	15	79	99	118	20	27	36	24	32	41
Smith(1969)	13	22	25	106	129	153	21	38	44	48	83	96
Wallis(1969)	14	22	24	99	128	156	25	39	46	43	55	69
Gregory & Scott(1969)	12	18	26	49	90	140	16	27	46	36	62	94
Premoli et al(1970)	15	21	24	129	164	177	30	43	45	48	62	83
Rouhani I(1970) <sup>4</sup>	13	22	25	107	180	189	17	36	43	35	75	92
Rouhani II(1970) <sup>4</sup>	9	14	21	43	92	129	4	15	33	34	73	88
Bonnecaze et al(1971)	3	14	23	90	139	181	9	18	40	33	60	85
Dix(1971)	16	23	25	141	167	188	26	41	43	56	65	73
Beggs(1972)	19	25	25	79	116	146	26	40	43	37	57	71
Chisholm(1973)	17	25	26	101	126	146	24	41	46	58	91	96
Loscher & Reinhardt(1973)	7	8	10	59	90	99	26	35	42	36	41	50
Mattar & Gregory(1974)	0	0	0	0	3	43	0	0	2	0	1	19
Moussali <sup>5</sup>	6	11	18	35	56	80	16	27	35	24	47	65
Greskovich & Cooper(1975)	11	20	26	45	84	139	24	35	41	42	70	89
Madsen(1975)	0	0	0	0	4	5	0	0	0	0	0	0
Mukherjee(1979)	16	24	25	74	137	162	28	37	43	32	55	68
Gardner-1(1980)	4	7	13	52	77	95	7	14	23	23	30	36
Gardner-2(1980)	3	12	14	66	95	114	30	36	37	26	31	36
Sun et al(1980)	2	13	19	84	132	168	12	27	45	20	50	75
Jowitt(1981)	0	0	0	30	75	134	1	1	3	9	28	60
Chisholm(1983) - Armand(1946)	16	24	26	105	126	146	24	39	45	58	90	96
Spedding & Chen(1984)	8	12	15	101	125	159	23	28	33	40	55	62
Bestion(1985)	6	10	15	30	65	84	9	28	37	42	57	71
Tandon et al(1985)	8	15	16	69	107	121	21	29	34	22	35	36
Chen(1986)	10	18	22	103	129	146	26	36	45	47	66	75
Hamersma & Hart(1987)	4	7	7	33	53	68	9	20	27	14	21	27



Table 20. Number of data points correctly predicted for vertical flow by data set  
(contd.)

Correlation / Data Source	Beggs (1972) (Air -Water)			Spedding & Nguyen (1976) (Air -Water)			Mukherjee (1979) (Air -Kerosene)			Sujumnong (1997) (Air -Water)		
	Total Points 26			Total Points 224			Total Points 52			Total Points 101		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%
Kawaji et al(1987)	7	12	14	52	93	115	18	29	36	27	39	48
Minami & Brill(1987)	19	24	26	71	131	163	29	41	44	28	53	68
El-Boher et al(1988)	9	20	23	140	166	187	11	34	42	35	49	58
Hart et al(1989)	9	12	15	84	102	113	22	28	34	27	30	34
Kokal & Stanislav(1989)	3	14	23	89	139	179	9	18	40	33	60	85
Spedding et al(1989)	8	12	15	91	109	133	23	32	36	33	43	46
Toshiba(1989)	12	24	24	105	186	201	19	43	45	48	89	96
Huq & Loth(1992)	12	17	25	112	139	157	20	36	43	43	72	91
Inoue et al(1993)	9	17	23	95	132	142	26	37	43	51	77	85
Czop et al(1994)	3	8	14	39	84	122	9	13	31	16	24	42
AbdulMajeed(1996)	11	16	19	63	110	124	17	29	35	21	41	49
Maier and Coddington(1997)	7	13	15	90	125	134	26	35	39	33	50	53
Petalaz & Aziz(1997)	1	3	6	9	18	24	0	2	2	4	8	13
Gomez et al(2000)	0	0	1	49	77	87	0	1	2	14	19	25
Zhao et al(2000)	0	3	3	26	37	49	0	0	0	6	13	17
Graham et al(2001)	14	16	24	96	124	146	23	34	40	40	50	59
Spedding et al(1989)	8	12	15	91	109	133	23	32	36	33	43	46
Toshiba(1989)	12	24	24	105	186	201	19	43	45	48	89	96
Huq & Loth(1992)	12	17	25	112	139	157	20	36	43	43	72	91
Inoue et al(1993)	9	17	23	95	132	142	26	37	43	51	77	85
Czop et al(1994)	3	8	14	39	84	122	9	13	31	16	24	42
AbdulMajeed(1996)	11	16	19	63	110	124	17	29	35	21	41	49
Maier and Coddington(1997)	7	13	15	90	125	134	26	35	39	33	50	53
Petalaz & Aziz(1997)	1	3	6	9	18	24	0	2	2	4	8	13
Gomez et al(2000)	0	0	1	49	77	87	0	1	2	14	19	25
Zhao et al(2000)	0	3	3	26	37	49	0	0	0	6	13	17
Graham et al(2001)	14	16	24	96	124	146	23	34	40	40	50	59

<sup>1</sup> correlation as reported by Leung (2005), <sup>2,5</sup> correlations as reported by Kataoka and Ishii (1987), <sup>3</sup> correlation as reported by Isbin and Biddle (1979) , <sup>4</sup> Rouhani and Axelsson(1970) correlations I and II.

The Rouhani I(1970) correlation gave an excellent prediction all in all and has a noticeable improvement from its performance for the other inclined data sets where it had excessive over predictions in the lower void fraction range. Filimonov et al. (1957) also has good uniformity in its prediction with slight under predictions for void fractions less than 0.7. The Hughmark (1962) correlation over predicted the data while Dix (1971) gave a slight random scatter for the lower void fraction range. The excellent performance by Dix (1971) for the very high void fraction range should be well noted where measurement of experimental data is usually very difficult. Premoli et al. (1970) repeated its tendency to over predict the data as observed for the other inclination angles though there has been a significant improvement for the

Table 21. Percentage of data points correctly predicted for vertical flow by data set

Correlation / Data Source	Beggs (1972) (Air -Water)			Spedding & Nguyen (1976) (Air -Water)			Mukherjee (1979) (Air -Kerosene)			Sujumnong (1997) (Air -Water)		
	Total Points 26			Total Points 224			Total Points 52			Total Points 101		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%
Homogeneous	19.2	34.6	53.8	15.2	24.6	35.3	25.0	51.9	63.5	23.8	39.6	58.4
Armand(1946)	46.2	69.2	88.5	21.0	37.5	60.3	30.8	48.1	88.5	34.7	59.4	84.2
Armand - Massina <sup>1</sup>	65.4	92.3	96.2	31.3	58.5	65.6	48.1	76.9	90.4	43.6	76.2	91.1
Lockhart & Martinelli(1949)	50.0	76.9	96.2	42.4	56.3	67.4	42.3	63.5	84.6	45.5	55.4	62.4
Sterman(1956)	23.1	38.5	42.3	21.0	32.6	40.2	11.5	42.3	50.0	14.9	25.7	32.7
Filimonov et al(1957)	73.1	92.3	92.3	34.4	60.7	80.4	63.5	80.8	86.5	37.6	68.3	88.1
Chisholm & Laird(1958)	15.4	23.1	26.9	14.7	23.7	30.4	7.7	36.5	50.0	12.9	20.8	25.7
Flanigan(1958)	7.7	19.2	34.6	25.0	36.2	45.1	1.9	17.3	36.5	25.7	34.7	39.6
Dimentiev et al <sup>2</sup>	26.9	34.6	50.0	20.1	35.3	44.2	21.2	59.6	63.5	20.8	29.7	36.6
Hoogendroon(1959)	0.0	15.4	34.6	28.1	45.5	52.2	7.7	17.3	48.1	43.6	63.4	74.3
Bankoff(1960)	0.0	7.7	19.2	14.3	24.1	36.2	0.0	0.0	23.1	2.0	6.9	30.7
Fauske(1961)	26.9	26.9	30.8	18.3	27.2	31.7	19.2	46.2	50.0	18.8	23.8	25.7
Wilson et al(1961,1965)	11.5	26.9	38.5	7.6	16.5	26.8	11.5	23.1	34.6	3.0	5.0	12.9
Hughmark(1962)	38.5	92.3	96.2	56.7	71.4	75.4	30.8	75.0	88.5	68.3	78.2	88.1
Nicklin et al(1962)	11.5	53.8	88.5	40.2	62.5	80.8	17.3	34.6	76.9	32.7	59.4	84.2
Nishino & Yamazaki(1963)	34.6	57.7	65.4	48.2	62.9	74.6	44.2	59.6	73.1	38.6	46.5	51.5
Fujie(1964)	26.9	57.7	76.9	17.4	31.3	49.6	30.8	59.6	73.1	31.7	57.4	78.2
Kowalczewski(1964)	38.5	50.0	57.7	44.2	52.7	61.2	42.3	65.4	71.2	36.6	48.5	51.5
Thom(1964)	34.6	50.0	61.5	27.7	49.1	56.7	36.5	53.8	67.3	19.8	36.6	46.5
Zivi(1964)	34.6	46.2	53.8	36.6	45.5	52.7	44.2	59.6	71.2	27.7	39.6	41.6
Hughmark(1965)	34.6	65.4	88.5	17.4	36.6	55.4	26.9	42.3	73.1	34.7	55.4	78.2
Neal & Bankoff(1965)	11.5	19.2	23.1	10.7	20.1	29.9	11.5	15.4	17.3	5.9	10.9	19.8
Turner & Wallis(1965)	11.5	19.2	26.9	10.7	17.4	26.8	9.6	11.5	25.0	6.9	12.9	21.8
Baroczy(1966)	34.6	53.8	57.7	42.9	55.4	67.0	48.1	59.6	71.2	36.6	47.5	48.5
Guzhov et al(1967)	0.0	0.0	0.0	0.0	0.0	9.4	0.0	0.0	0.0	0.0	0.0	4.0
Kutucuglu <sup>3</sup>	23.1	46.2	57.7	35.3	44.2	52.7	38.5	51.9	69.2	23.8	31.7	40.6
Smith(1969)	50.0	84.6	96.2	47.3	57.6	68.3	40.4	73.1	84.6	47.5	82.2	95.0
Wallis(1969)	53.8	84.6	92.3	44.2	57.1	69.6	48.1	75.0	88.5	42.6	54.5	68.3
Gregory & Scott(1969)	46.2	69.2	100.0	21.9	40.2	62.5	30.8	51.9	88.5	35.6	61.4	93.1
Premoli et al(1970)	57.7	80.8	92.3	57.6	73.2	79.0	57.7	82.7	86.5	47.5	61.4	82.2
Rouhani I(1970) <sup>4</sup>	50.0	84.6	96.2	47.8	80.4	84.4	32.7	69.2	82.7	34.7	74.3	91.1
Rouhani II(1970) <sup>4</sup>	34.6	53.8	80.8	19.2	41.1	57.6	7.7	28.8	63.5	33.7	72.3	87.1
Bonnecaze et al(1971)	11.5	53.8	88.5	40.2	62.1	80.8	17.3	34.6	76.9	32.7	59.4	84.2
Dix(1971)	61.5	88.5	96.2	62.9	74.6	83.9	50.0	78.8	82.7	55.4	64.4	72.3
Beggs(1972)	73.1	96.2	96.2	35.3	51.8	65.2	50.0	76.9	82.7	36.6	56.4	70.3
Chisholm(1973)	65.4	96.2	100.0	45.1	56.3	65.2	46.2	78.8	88.5	57.4	90.1	95.0
Loscher & Reinhardt(1973)	26.9	30.8	38.5	26.3	40.2	44.2	50.0	67.3	80.8	35.6	40.6	49.5
Mattar & Gregory(1974)	0.0	0.0	0.0	0.0	1.3	19.2	0.0	0.0	3.8	0.0	1.0	18.8
Moussali <sup>5</sup>	23.1	42.3	69.2	15.6	25.0	35.7	30.8	51.9	67.3	23.8	46.5	64.4
Greskovich & Cooper(1975)	42.3	76.9	100.0	20.1	37.5	62.1	46.2	67.3	78.8	41.6	69.3	88.1
Madsen(1975)	0.0	0.0	0.0	0.0	1.8	2.2	0.0	0.0	0.0	0.0	0.0	0.0
Mukherjee(1979)	61.5	92.3	96.2	33.0	61.2	72.3	53.8	71.2	82.7	31.7	54.5	67.3
Gardner-1(1980)	15.4	26.9	50.0	23.2	34.4	42.4	13.5	26.9	44.2	22.8	29.7	35.6
Gardner-2(1980)	11.5	46.2	53.8	29.5	42.4	50.9	57.7	69.2	71.2	25.7	30.7	35.6
Sun et al(1980)	7.7	50.0	73.1	37.5	58.9	75.0	23.1	51.9	86.5	19.8	49.5	74.3
Jowitt(1981)	0.0	0.0	0.0	13.4	33.5	59.8	1.9	1.9	5.8	8.9	27.7	59.4
Chisholm(1983) - Armand(1946)	61.5	92.3	100.0	46.9	56.3	65.2	46.2	75.0	86.5	57.4	89.1	95.0
Spedding & Chen(1984)	30.8	46.2	57.7	45.1	55.8	71.0	44.2	53.8	63.5	39.6	54.5	61.4
Bestion(1985)	23.1	38.5	57.7	13.4	29.0	37.5	17.3	53.8	71.2	41.6	56.4	70.3
Tandon et al(1985)	30.8	57.7	61.5	30.8	47.8	54.0	40.4	55.8	65.4	21.8	34.7	35.6
Chen(1986)	38.5	69.2	84.6	46.0	57.6	65.2	50.0	69.2	86.5	46.5	65.3	74.3
Hamersma & Hart(1987)	15.4	26.9	26.9	14.7	23.7	30.4	17.3	38.5	51.9	13.9	20.8	26.7

Table 21. Percentage of data points correctly predicted for vertical flow by data set  
(contd.)

Correlation / Data Source	Beggs (1972)			Spedding & Nguyen (1976)			Mukherjee (1979)			Sujumnong (1997)		
	(Air -Water)			(Air -Water)			(Air -Kerosene)			(Air -Water)		
	Total Points 26			Total Points 224			Total Points 52			Total Points 101		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%	±5%	±10%	±15%
Kawaji et al(1987)	26.9	46.2	53.8	23.2	41.5	51.3	34.6	55.8	69.2	26.7	38.6	47.5
Minami & Brill(1987)	73.1	92.3	100.0	31.7	58.5	72.8	55.8	78.8	84.6	27.7	52.5	67.3
El-Boher et al(1988)	34.6	76.9	88.5	62.5	74.1	83.5	21.2	65.4	80.8	34.7	48.5	57.4
Hart et al(1989)	34.6	46.2	57.7	37.5	45.5	50.4	42.3	53.8	65.4	26.7	29.7	33.7
Kokal & Stanislav(1989)	11.5	53.8	88.5	39.7	62.1	79.9	17.3	34.6	76.9	32.7	59.4	84.2
Spedding et al(1989)	30.8	46.2	57.7	40.6	48.7	59.4	44.2	61.5	69.2	32.7	42.6	45.5
Toshiba(1989)	46.2	92.3	92.3	46.9	83.0	89.7	36.5	82.7	86.5	47.5	88.1	95.0
Huq & Loth(1992)	46.2	65.4	96.2	50.0	62.1	70.1	38.5	69.2	82.7	42.6	71.3	90.1
Inoue et al(1993)	34.6	65.4	88.5	42.4	58.9	63.4	50.0	71.2	82.7	50.5	76.2	84.2
Czop et al(1994)	11.5	30.8	53.8	17.4	37.5	54.5	17.3	25.0	59.6	15.8	23.8	41.6
AbdulMajeed(1996)	42.3	61.5	73.1	28.1	49.1	55.4	32.7	55.8	67.3	20.8	40.6	48.5
Maier and Coddington(1997)	26.9	50.0	57.7	40.2	55.8	59.8	50.0	67.3	75.0	32.7	49.5	52.5
Petalaz & Aziz(1997)	3.8	11.5	23.1	4.0	8.0	10.7	0.0	3.8	3.8	4.0	7.9	12.9
Gomez et al(2000)	0.0	0.0	3.8	21.9	34.4	38.8	0.0	1.9	3.8	13.9	18.8	24.8
Zhao et al(2000)	0.0	11.5	11.5	11.6	16.5	21.9	0.0	0.0	0.0	5.9	12.9	16.8
Graham et al(2001)	53.8	61.5	92.3	42.9	55.4	65.2	44.2	65.4	76.9	39.6	49.5	58.4

<sup>1</sup> correlation as reported by Leung (2005), <sup>2,5</sup> correlations as reported by Kataoka and Ishii (1987), <sup>3</sup> correlation as reported by Isbin and Biddle (1979), <sup>4</sup> Rouhani and Axelsson(1970) correlations I and II.

vertical case. It also gave an excellent result for the higher void fraction range comparable to Dix (1971).

The combined air water data of Beggs (1972), Spedding and Nguyen (1976) and Sujumnong (1997) is well captured by Toshiba (1989) that predicted 81.5% of the data within the 15% index. The correlations of Rouhani I (1970) and Filimonov et al. (1957) gave comparably close predictions to that of Toshiba (1989). The next best predictions are from Dix (1971), Premoli et al. (1970) and Hughmark (1962) which gave results of 81.5, 80.9 and 80.6% for the same error index.

Owing to the small number of data in the air kerosene vertical data set of Mukherjee (1979), fifteen correlations were able to predict 85% and above for the 15% error percentage. Armand Massina (Leung 2005) gave 90.4% while Chisholm (1973) and Armand (1946) predicted 88.5% each, respectively. The correlations by Filimonov et al. (1957), Premoli et al. (1970), Sun et al. (1980) Chisholm (1983) – Armand (1946),

Chen (1986) and Toshiba (1989) all predicted 86.5% of the data sets within the wider error band of 15%.

#### **4.4 Summarized comparison and discussion for all data sets**

We have presented an extensive comparison of the predictive capability of the different void fraction correlations with experimental data for horizontal, inclined and vertical flow cases. Recommendations and conclusions were drawn for each section with regard to the performance of the top performing correlations. The top six correlations which predicted about 85% of the whole data base within the 15% error index were selected and their consistency across the void fraction spectrum presented. The total number and percentage of data points correctly predicted for the whole data base by each correlation is given in Table 23.

Based on the analysis for the three scenarios considered, six correlations are recommended for acceptably predicting void fraction for horizontal and upward inclined pipes regardless of flow regimes.

Basing our judgment of the capability of the correlations only on the total number of data points correctly predicted, we can have two groups of correlations where the three correlations within each group have very close performance to one another.

The first group of correlations consists of Toshiba (1989), Rouhani I(1970) and Dix(1971) with 85.3,84.2 and 83.1 %,respectively, prediction of the total points within the 15% error index. The second group is comprised of Hughmark (1962), Premoli et al. (1970) and Filimonov et al. (1957) with 81.6,81 and 80.6 %, respectively, of the total points correctly captured within the 15% error index.

Hence, the correlation by Toshiba (1989) has shown best prediction capability in the inclined and vertical flow cases with a very good agreement with the horizontal experimental data set making it the top correlation to be worth a general

Table 22. Total number and percentage of data points correctly predicted for Vertical flow database (all sources)

Correlation / Data Source	Total data points correctly predicted for the whole database			Percentage of data points correctly predicted for the whole database		
	Total Points 403			Total Points 403		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%
Homogeneous	76	131	185	18.9	32.5	45.9
Armand(1946)	110	187	289	27.3	46.4	71.7
Armand - Massina <sup>1</sup>	156	272	311	38.7	67.5	77.2
Lockhart & Martinelli(1949)	176	235	283	43.7	58.3	70.2
Sterman(1956)	74	131	160	18.4	32.5	39.7
Filimonov et al(1957)	167	271	338	41.4	67.2	83.9
Chisholm & Laird(1958)	54	99	127	13.4	24.6	31.5
Flanigan(1958)	85	130	169	21.1	32.3	41.9
Dimentiev et al <sup>2</sup>	84	149	182	20.8	37.0	45.2
Hoogendroon(1959)	111	179	226	27.5	44.4	56.1
Bankoff(1960)	34	63	129	8.4	15.6	32.0
Fauske(1961)	77	116	131	19.1	28.8	32.5
Wilson et al(1961,1965)	29	61	101	7.2	15.1	25.1
Hughmark(1962)	222	302	329	55.1	74.9	81.6
Nicklin et al(1962)	135	232	329	33.5	57.6	81.6
Nishino & Yamazaki(1963)	179	234	274	44.4	58.1	68.0
Fujie(1964)	94	174	248	23.3	43.2	61.5
Kowalczewski(1964)	168	214	241	41.7	53.1	59.8
Thom(1964)	110	188	225	27.3	46.7	55.8
Zivi(1964)	142	185	211	35.2	45.9	52.4
Hughmark(1965)	97	177	264	24.1	43.9	65.5
Neal & Bankoff(1965)	39	69	102	9.7	17.1	25.3
Turner & Wallis(1965)	39	63	102	9.7	15.6	25.3
Baroczy(1966)	167	217	251	41.4	53.8	62.3
Guzhov et al(1967)	0	0	25	0.0	0.0	6.2
Kutucuglu <sup>3</sup>	129	170	210	32.0	42.2	52.1
Smith(1969)	188	272	318	46.7	67.5	78.9
Wallis(1969)	181	244	295	44.9	60.5	73.2
Gregory & Scott(1969)	113	197	306	28.0	48.9	75.9
Premoli et al(1970)	222	290	329	55.1	72.0	81.6
Rouhani I(1970) <sup>4</sup>	172	313	349	42.7	77.7	86.6
Rouhani II(1970) <sup>4</sup>	90	194	271	22.3	48.1	67.2
Bonnecaze et al(1971)	135	231	329	33.5	57.3	81.6
Dix(1971)	239	296	329	59.3	73.4	81.6
Beggs(1972)	161	238	285	40.0	59.1	70.7
Chisholm(1973)	200	283	314	49.6	70.2	77.9
Loscher & Reinhardt(1973)	128	174	201	31.8	43.2	49.9
Mattar & Gregory(1974)	0	4	64	0.0	1.0	15.9
Moussali <sup>5</sup>	81	141	198	20.1	35.0	49.1
Greskovich & Cooper(1975)	122	209	295	30.3	51.9	73.2
Madsen(1975)	0	4	5	0.0	1.0	1.2
Mukherjee(1979)	150	253	298	37.2	62.8	73.9
Gardner-1(1980)	86	128	167	21.3	31.8	41.4
Gardner-2(1980)	125	174	201	31.0	43.2	49.9

Table 22. Total number and percentage of data points correctly predicted for Vertical flow database (all sources) (contd.)

Correlation / Data Source	Total data points correctly predicted for the whole database			Percentage of data points correctly predicted for the whole database		
	Total Points 403			Total Points 403		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%
Sun et al(1980)	118	222	307	29.3	55.1	76.2
Jowitt(1981)	40	104	197	9.9	25.8	48.9
Chisholm(1983) - Armand(1946)	203	279	313	50.4	69.2	77.7
Spedding & Chen(1984)	172	220	269	42.7	54.6	66.7
Bestion(1985)	87	160	207	21.6	39.7	51.4
Tandon et al(1985)	120	186	207	29.8	46.2	51.4
Chen(1986)	186	249	288	46.2	61.8	71.5
Hamersma & Hart(1987)	60	101	129	14.9	25.1	32.0
Kawaji et al(1987)	104	173	213	25.8	42.9	52.9
Minami & Brill(1987)	147	249	301	36.5	61.8	74.7
El-Boher et al(1988)	195	269	310	48.4	66.7	76.9
Hart et al(1989)	142	172	196	35.2	42.7	48.6
Kokal & Stanislav(1989)	134	231	327	33.3	57.3	81.1
Spedding et al(1989)	155	196	230	38.5	48.6	57.1
Toshiba(1989)	184	342	366	45.7	84.9	90.8
Huq & Loth(1992)	187	264	316	46.4	65.5	78.4
Inoue et al(1993)	181	263	293	44.9	65.3	72.7
Czop et al(1994)	67	129	209	16.6	32.0	51.9
AbdulMajeed(1996)	112	196	227	27.8	48.6	56.3
Maier and Coddington(1997)	156	223	241	38.7	55.3	59.8
Petalaz & Aziz(1997)	14	31	45	3.5	7.7	11.2
Gomez et al(2000)	63	97	115	15.6	24.1	28.5
Zhao et al(2000)	32	53	69	7.9	13.2	17.1
Graham et al(2001)	173	224	269	42.9	55.6	66.7

<sup>1</sup> correlation as reported by Leung (2005), <sup>2,5</sup> correlations as reported by Kataoka and Ishii (1987), <sup>3</sup> correlation as reported by Isbin and Biddle (1979), <sup>4</sup> Rouhani and Axelsson(1970) correlations I and II.

recommendation. The next on the list would be that of Rouhani I (1970) with a very close performance to that of Toshiba (1989). On top of the slight numerical percent advantage, its relatively good consistency across the whole void fraction range for the total data set has been given weight over the excellent consistency shown by Dix (1971) for the combined inclined experimental data set with a wide scatter for the horizontal data which is placed third. However, it should be pointed out that the Dix (1971) correlation has the best prediction with the tighter error indices of 5 and 10% above all of the correlations considered in this analysis.

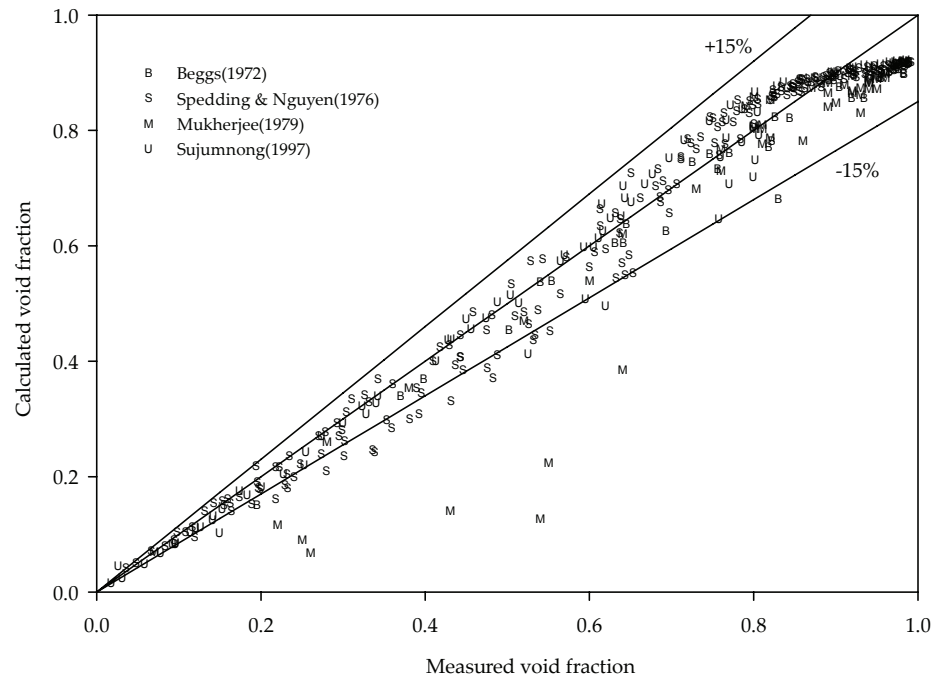


Figure 35. Comparison of Toshiba (1989) correlation with measured combined vertical experimental data

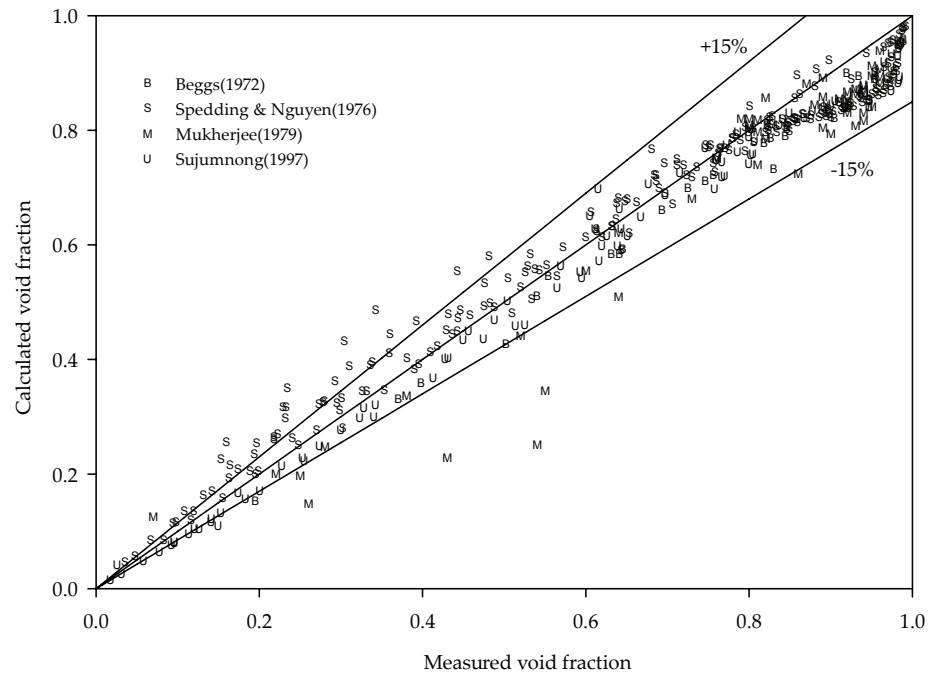


Figure 36. Comparison of Rouhani I (1970) correlation with measured combined vertical experimental data

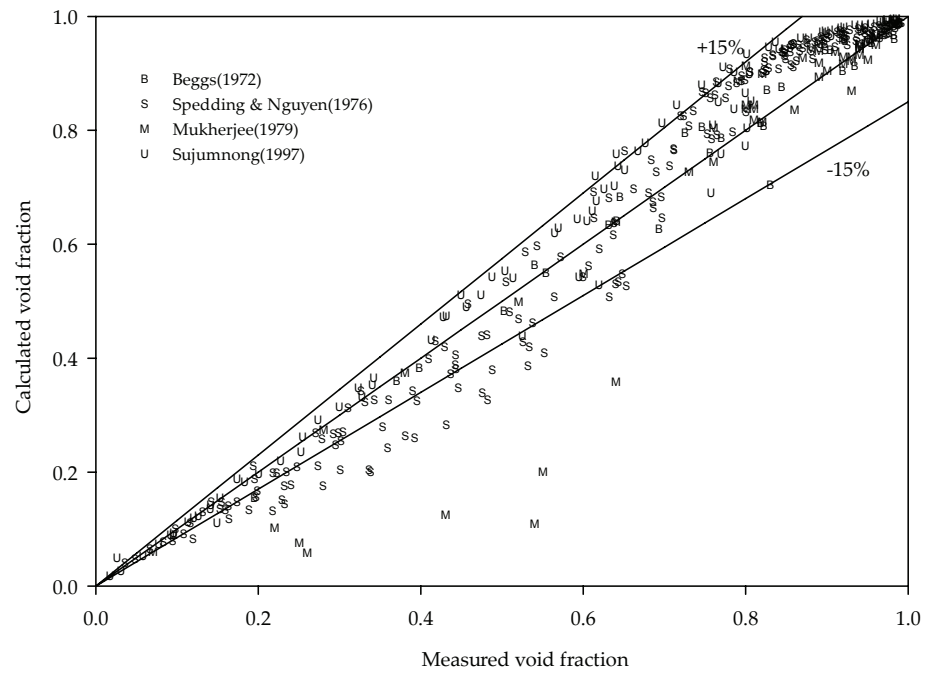


Figure 37. Comparison of Filimonov et al. (1957) correlation with measured combined vertical experimental data

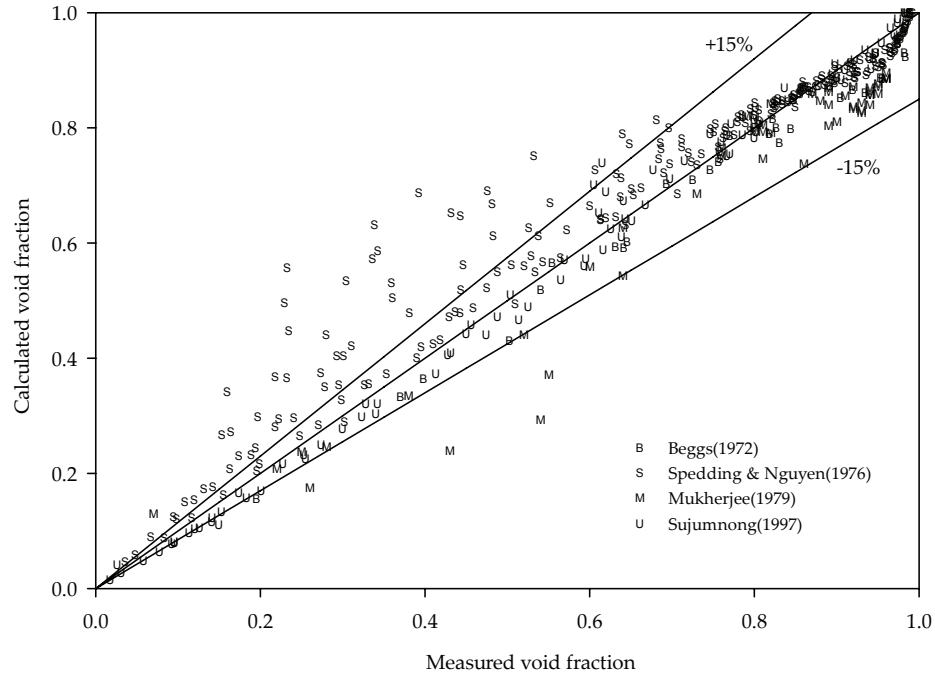


Figure 38. Comparison of Hughmark (1962) correlation with measured combined vertical experimental data



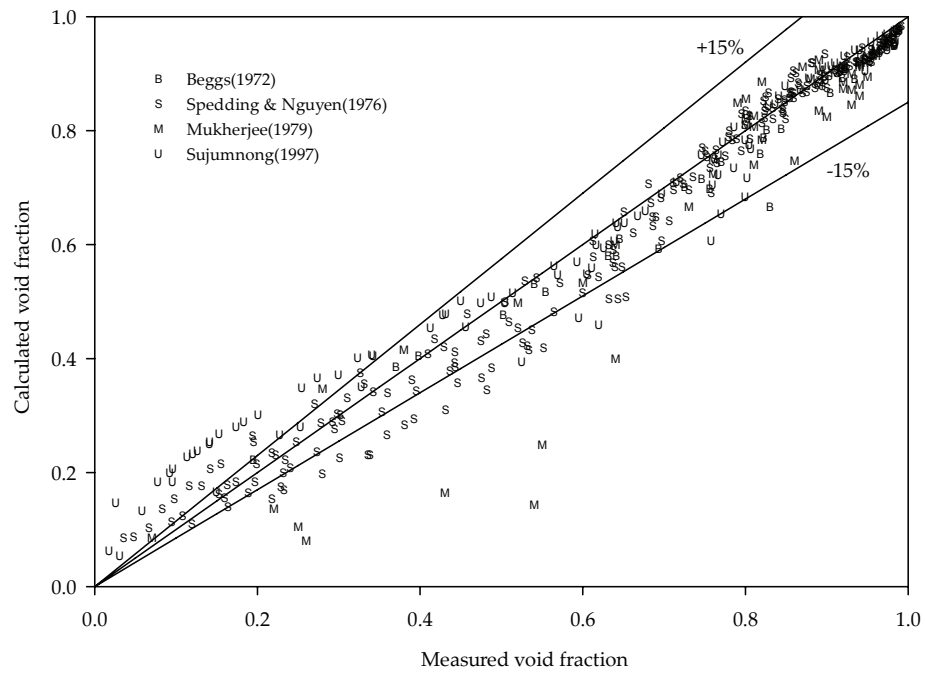


Figure 39. Comparison of Dix (1971) correlation with measured combined vertical experimental data

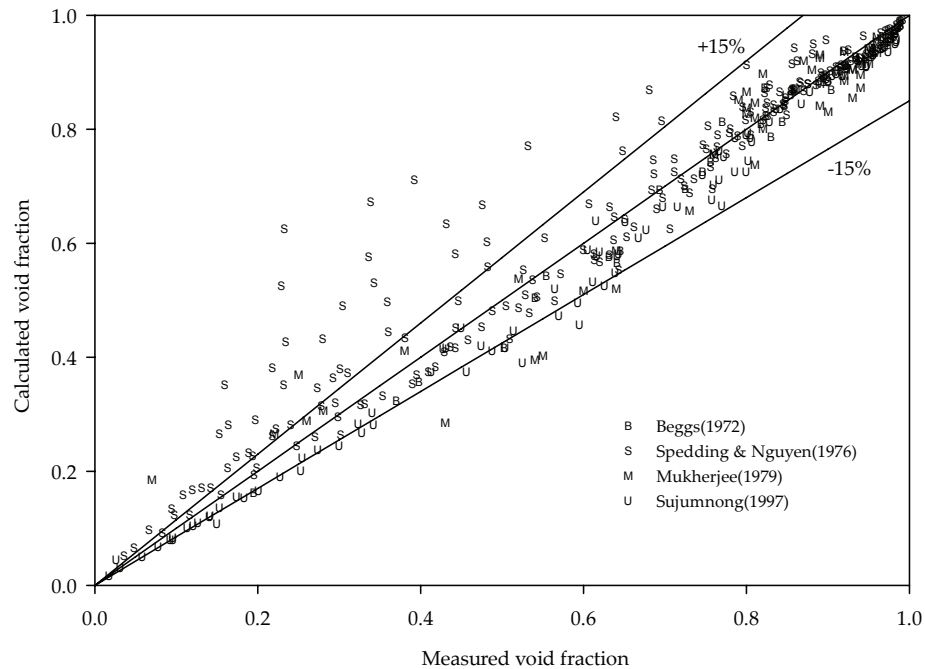


Figure 40. Comparison of Premoli et al. (1970) correlation with measured combined vertical experimental data

Table 23. Total number and percentage of data points correctly predicted for the whole database (all sources)

Correlation / Data Source	Total data points correctly predicted for the whole database			Percentage of data points correctly predicted for the whole database		
	Total Points 2845			Total Points 2845		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%
Homogeneous	688	1109	1449	24.2	39.0	50.9
Armand(1946)	657	1234	1952	23.1	43.4	68.6
Armand - Massina <sup>1</sup>	1205	1992	2259	42.4	70.0	79.4
Lockhart & Martinelli(1949)	1364	1852	2128	47.9	65.1	74.8
Sterman(1956)	585	927	1163	20.6	32.6	40.9
Filimonov et al(1957)	1369	1953	2294	48.1	68.6	80.6
Chisholm & Laird(1958)	532	828	1060	18.7	29.1	37.3
Flanigan(1958)	517	917	1231	18.2	32.2	43.3
Dimentiev et al <sup>2</sup>	656	1083	1386	23.1	38.1	48.7
Hoogendroon(1959)	557	1018	1424	19.6	35.8	50.1
Bankoff(1960)	215	467	869	7.6	16.4	30.5
Fauske(1961)	708	957	1093	24.9	33.6	38.4
Wilson et al(1961,1965)	264	545	768	9.3	19.2	27.0
Hughmark(1962)	1244	2003	2322	43.7	70.4	81.6
Nicklin et al(1962)	578	1211	1921	20.3	42.6	67.5
Nishino & Yamazaki(1963)	1194	1630	1899	42.0	57.3	66.7
Fujie(1964)	829	1387	1831	29.1	48.8	64.4
Kowalczewski(1964)	1179	1574	1758	41.4	55.3	61.8
Thom(1964)	946	1441	1692	33.3	50.7	59.5
Zivi(1964)	1076	1367	1565	37.8	48.0	55.0
Hughmark(1965)	601	1137	1788	21.1	40.0	62.8
Neal & Bankoff(1965)	277	485	679	9.7	17.0	23.9
Turner & Wallis(1965)	406	642	847	14.3	22.6	29.8
Baroczy(1966)	1129	1505	1762	39.7	52.9	61.9
Guzhov et al(1967)	5	71	214	0.2	2.5	7.5
Kutucuglu <sup>3</sup>	1022	1345	1554	35.9	47.3	54.6
Smith(1969)	1355	1925	2213	47.6	67.7	77.8
Wallis(1969)	1349	1842	2130	47.4	64.7	74.9
Gregory & Scott(1969)	679	1289	2104	23.9	45.3	74.0
Premoli et al(1970)	1643	2084	2304	57.8	73.3	81.0
Rouhani I(1970) <sup>4</sup>	1082	2059	2395	38.0	72.4	84.2
Rouhani II(1970) <sup>4</sup>	610	1273	1810	21.4	44.7	63.6
Bonnecaze et al(1971)	582	1211	1925	20.5	42.6	67.7
Dix(1971)	1597	2139	2363	56.1	75.2	83.1
Beggs(1972)	1304	1843	2141	45.8	64.8	75.3
Chisholm(1973)	1382	1959	2234	48.6	68.9	78.5
Loscher & Reinhardt(1973)	1188	1454	1638	41.8	51.1	57.6
Mattar & Gregory(1974)	40	141	417	1.4	5.0	14.7
Moussali <sup>5</sup>	758	1224	1583	26.6	43.0	55.6
Greskovich & Cooper(1975)	1027	1575	2014	36.1	55.4	70.8
Madsen(1975)	80	137	178	2.8	4.8	6.3
Mukherjee(1979)	1262	1872	2217	44.4	65.8	77.9
Gardner-1(1980)	642	1048	1353	22.6	36.8	47.6
Gardner-2(1980)	880	1324	1575	30.9	46.5	55.4

Table 23. Total number and percentage of data points correctly predicted for the whole database (all sources)(contd.)

Correlation / Data Source	Total data points correctly predicted for the whole database			Percentage of data points correctly predicted for the whole database		
	Total Points 2845			Total Points 2845		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%
Sun et al(1980)	590	1276	1960	20.7	44.9	68.9
Jowitt(1981)	235	522	1057	8.3	18.3	37.2
Chisholm(1983) - Armand(1946)	1349	1940	2237	47.4	68.2	78.6
Spedding & Chen(1984)	1172	1549	1855	41.2	54.4	65.2
Bestion(1985)	674	1155	1455	23.7	40.6	51.1
Tandon et al(1985)	1085	1500	1748	38.1	52.7	61.4
Chen(1986)	1339	1780	2022	47.1	62.6	71.1
Hamersma & Hart(1987)	571	873	1097	20.1	30.7	38.6
Kawaji et al(1987)	815	1315	1615	28.6	46.2	56.8
Minami & Brill(1987)	1315	1887	2208	46.2	66.3	77.6
El-Boher et al(1988)	1176	1808	2177	41.3	63.6	76.5
Hart et al(1989)	1191	1479	1649	41.9	52.0	58.0
Kokal & Stanislav(1989)	579	1208	1921	20.4	42.5	67.5
Spedding et al(1989)	1083	1433	1663	38.1	50.4	58.5
Toshiba(1989)	1065	2137	2427	37.4	75.1	85.3
Huq & Loth(1992)	1351	1888	2195	47.5	66.4	77.2
Inoue et al(1993)	1109	1709	2019	39.0	60.1	71.0
Czop et al(1994)	379	770	1347	13.3	27.1	47.3
AbdulMajeed(1996)	887	1526	1776	31.2	53.6	62.4
Maier and Coddington(1997)	1040	1527	1744	36.6	53.7	61.3
Petalaz & Aziz(1997)	108	224	346	3.8	7.9	12.2
Gomez et al(2000)	481	715	879	16.9	25.1	30.9
Zhao et al(2000)	297	441	550	10.4	15.5	19.3
Graham et al(2001)	1347	1810	2092	47.3	63.6	73.5

<sup>1</sup> correlation as reported by Leung (2005), <sup>2,5</sup> correlations as reported by Kataoka and Ishii (1987), <sup>3</sup> correlation as reported by Isbin and Biddle (1979), <sup>4</sup> Rouhani and Axelsson(1970) correlations I and II.

On the second group of correlations, though ranked lower than that Hughmark (1962) in Table 24 based on the 15 % error index for the whole data set, Premoli et al.(1970) correlation has the best capability in the tighter indices which is very comparable to that of Dix(1971). Moreover, the Premoli et al. (1970) correlation has predicted all the data bases with very good accuracy and is the second only correlation to do so behind Rouhani I (1970). Hence it would deserve a general recommendation above that of Hughmark (1962) and Filimonov(1957) within this group.

The dilemma to chose between the correlations of Rouhani I(1970) and Premoli et al.(1970) over Dix(1971) and Filimonov et al.(1957) respectively could be resolved if

Table 24. Total number and percentage of data points correctly predicted by the top six correlations for the whole database (all sources)

Correlation	Total data points correctly predicted for the whole database			Percentage of data points correctly predicted for the whole database		
	Total Points 2845			Total Points 2845		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%
Toshiba(1989)	1065	2137	2427	37.4	75.1	85.3
Rouhani I(1970) <sup>1</sup>	1082	2059	2395	38.0	72.4	84.2
Dix(1971)	1597	2139	2363	56.1	75.2	83.1
Hughmark(1962)	1244	2003	2322	43.7	70.4	81.6
Premoli et al.(1970)	1643	2084	2304	57.8	73.3	81.0
Filimonov et al.(1957)	1369	1953	2294	48.1	68.6	80.6

<sup>1</sup> Rouhani and Axelsson(1970) correlations I.

one is interested to predict void fraction in inclined flow only. For this case the superior performances of the latter two correlations would out weigh the general applicability of the former ones.

In addition to the correlations discussed above, the correlations of Chisholm (1973), Chisholm (1983) – Armand (1946) and Armand-Massina (Leung 2005) have also given reasonable prediction for the whole data base with quite a very good performance for horizontal data sets. Their performances are very much close to one another which is to be expected as they are combinations and/or derivations of each other.

The scatter plots for the correlations of Toshiba (1989),Rouhani I (1970),Dix (1971), Premoli et al. (1970), Hughmark (1962) and Filimonov et al. (1957) with the whole data set are given in Figures 41 to 46.

We have mentioned the general failure of all the correlations in predicting any of the experimental data within the 5% index. The accuracy of the data set within this range is also one issue which we have raised as open to question and may not at all be the problem of the correlations.

To make a conclusive statement on this and also in trying to predict the compiled experimental data base with greater accuracy than what we have achieved so far, an effort is made to modify one of the best performing correlations so as to remove some of its weaknesses.

The next section would discuss the details of how this has been undertaken.

### **MODIFIED CORRELATION**

The drift flux analysis given by Zuber and Findlay (1965) has been proven to be quite a powerful tool to correlate experimental data as well as develop void fraction correlations. Of the six best performing void fraction correlations we have recommended all except the correlation of Hughmark (1962) are developed based on that model. It is to be recalled that this types of correlations have the general expression of the form

$$\alpha = \frac{U_{SG}}{C_0 U_M + U_{GM}}$$

where  $C_0$  is the distribution parameter and  $U_{GM} = U_G - U_M$  is the drift velocity.

A decision was made to make improvements on the top performing correlations so as to overcome some of the weaknesses observed in the analysis. It is worth mentioning here that the correlations of Rouhani I (1970) and Dix (1971) share similar parameters in their correlations. They differ in their expression for the distribution parameter while they share exactly the same expression for their drift velocity parameter with a very minor difference in the constant term.

Considering the fact that the Dix (1971) correlation has an outstanding consistency in all its predictions over the entire void fraction range and the fact that it has the highest prediction of data points within the tighter 5% error index makes it a prime candidate for further improvement.

The correlation of Dix (1971) and all the other correlations have significantly underperformed for the data of Spedding and Nguyen (1976). A close look at the data reveals that the system pressure is quite different from that of all the other data sets considered here. The prediction of the correlations further deteriorates with inclination angle for this data set. The other variable with this data set is the slightly “off standard” pipe diameter used in their experiment. Using these inputs, a damping correction factor is suggested which takes into account some of the observations made in previous void fraction developments. These are:

The drift velocity in the two phase slug flow which is predominantly encountered in horizontal as well as upward inclined two phase flow applications has been shown by Zuber and Findlay(1965) and many others to have the general form

$$U_{GM} = C \left[ \frac{g(\rho_L - \rho_G)D}{\rho_L^n} \right]^m$$

where  $C$ ,  $m$  and  $n$  are constants which have been shown to be flow pattern dependent.

An observation of the Dix (1971) correlation would reveal that the correlation is more geared to the “churn-turbulent bubbly flow” correlation suggested by Zuber and Findlay (1965). Considering the fact that slug flow is common in horizontal and upward inclined flows, the diameter term is introduced to take care of that as could be found in many of the correlations developed for slug flow.

The inclination angle and the system pressure effect on the measured void fraction data were closely analyzed. As outlined by Zuber and Findlay (1965) the drift velocity is a function of the concentration profiles and also depends on the momentum transfer between the two phases. Bankoff (1960) has shown that the non uniformity in phase distribution is a function of pressure while Zuber and Findlay (1965) has stated that

the drift velocity is a function of the concentration profiles and also depends on the momentum transfer between the phases. The concentration profiles across the pipe for a given inclination are generally assumed constant for flows without mass transfer along the pipe length. Integrating these observations and noting that this effect is normal to the pipe inclination, a correction factor of the form

$$[1 + \cos \theta]^{0.25}$$

is introduced.

The functional dependency of drift velocity on the momentum transfer between the phases assuming one dimensional force interaction would be a maximum for pipes with very high inclination angles operating near atmospheric pressure. This effect could conveniently be captured by the factor

$$[1 + \sin \theta]^{\frac{P_{atm}}{P_{system}}}$$

where  $P_{atm}$  and  $P_{system}$  are the atmospheric and system pressures respectively. These two correction factors have been introduced into the drift velocity expression of Dix (1971) correlation.

Hence the combined drift velocity expression for the modified correlation is,

$$U_{GM} = 2.9 \left[ \frac{gD\sigma(1 + \cos \theta)(\rho_L - \rho_G)}{\rho_L^2} \right]^{0.25} (1.22 + 1.22 \sin \theta)^{\frac{P_{atm}}{P_{system}}}$$

while the distribution coefficient,  $C_0$ , being the same as given by Dix(1971). This correlation would be referred to as the present study correlation and is given as

$$\alpha = \frac{U_{SG}}{U_{SG} \left( 1 + \left( \frac{U_{SL}}{U_{SG}} \right)^{\left( \frac{\rho_G}{\rho_L} \right)^{0.1}} \right) + 2.9 \left[ \frac{gD\sigma(1 + \cos \theta)(\rho_L - \rho_G)}{\rho_L^2} \right]^{0.25} (1.22 + 1.22 \sin \theta)^{\frac{P_{atm}}{P_{system}}}}$$

The performance of the top ranked Toshiba(1989) correlation could also be improved by replacing the constant drift velocity value with our modified expression given above while retaining the near uniform constant distribution coefficient,  $C_0 = 1.08$ , given by Toshiba(1989) which is seen to better capture the inclined data at lower void fraction quite well. The detail discussion of this correlation has been omitted here as the inherent consistency problem associated with the fixed value taken for the distribution parameter has been considered a weak point as compared to the present study correlation we have suggested here.

The performance of the present study correlation on the whole database in comparison to the best performing correlations is presented in Table 25 while the scatter plot for the whole data set is presented in Figure 47. It can be seen that the under prediction by the original Dix (1971) correlation on a substantial number of horizontal, inclined and vertical data points has been improved.

The excessively under predicted twelve horizontal data points which are also common to the prediction of Toshiba(1989), Rouhani I(1970), Dix(1971) and Filimonov et al.(1957) were further scrutinized individually.

These data points coincidentally come from the horizontal data of Spedding and Nguyen (1976) with common characteristics like,

1. All of them are the first data points to be measured within their group
2. Their flow regime was reported to be stratified
3. They have excessively low air mass flow rates in comparison to their next neighbors with jumps ranging from 237% to 853% while the water mass flow rate is literally unchanged
4. The void fraction to liquid hold up ratio reported was the least within their group which is quite expected.



Table 25. Total number and percentage of data points correctly predicted by the top six and modified correlation for the whole database (all sources)

Correlation	Total data points correctly predicted for the whole database			Percentage of data points correctly predicted for the whole database		
	Total Points 2845			Total Points 2845		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%
Toshiba(1989)	1065	2137	2427	37.4	75.1	85.3
Rouhani I(1970) <sup>1</sup>	1082	2059	2395	38.0	72.4	84.2
Dix(1971)	1597	2139	2363	56.1	75.2	83.1
Hughmark(1962)	1244	2003	2322	43.7	70.4	81.6
Premoli et al.(1970)	1643	2084	2304	57.8	73.3	81.0
Filimonov et al.(1957)	1369	1953	2294	48.1	68.6	80.6
Present Study	1718	2234	2436	60.4	78.5	85.6

<sup>1</sup> Rouhani and Axelsson(1970) correlations I.

In addition to the fact that measuring the liquid holdup for such a situation where the liquid phase is very small in comparison to the test section volume can prove difficult, the inaccuracy of the presented data for these points may have also arisen when the erroneous assumption that whatever volume is left of the drained liquid is taken as the gas volume fraction. This would not be valid as the flow being considered here is nearly a single phase(liquid) flow with very small volume flow rate which would translate into a small volume of liquid being trapped between the sections. The remaining portion of the test section already occupied with air even without starting the test run may have been wrongly interpreted as the void fraction in the final computations which is not reflective of the input conditions. The case is seen to improve as the mass flow rate of air is increased hence minimizing the difference between the actual volumes occupied by the “test” air with that of the preexisting one. Therefore, without additional information as to how the gas fraction was calculated, it was concluded that the data for all cases where the air mass flow rate is almost negligible is very questionable. Hence, discarding these data sets is justifiable and the predictions of all the correlations should be seen from this perspective. The present

study correlation after removing the 12 “irregular” data points is presented in Figure 48.

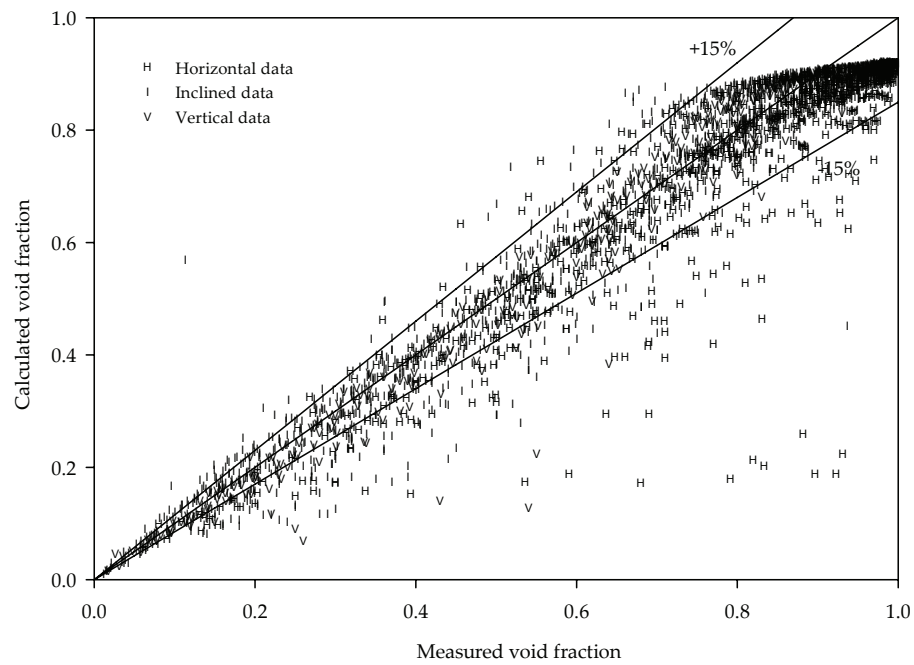


Figure 41. Comparison of Toshiba (1989) correlation with measured combined total experimental data

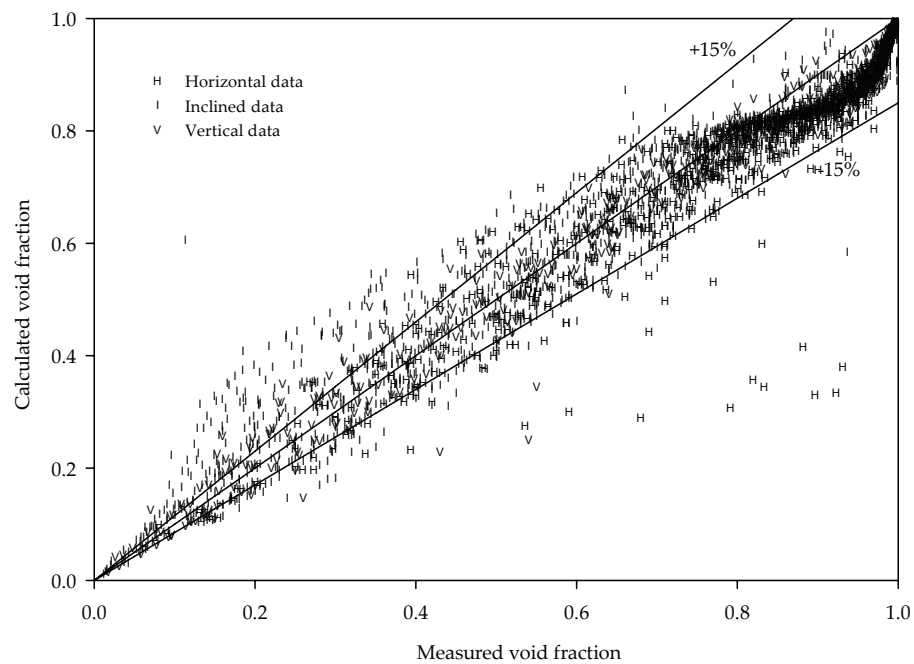


Figure 42. Comparison of Rouhani I (1970) correlation with measured combined total experimental data

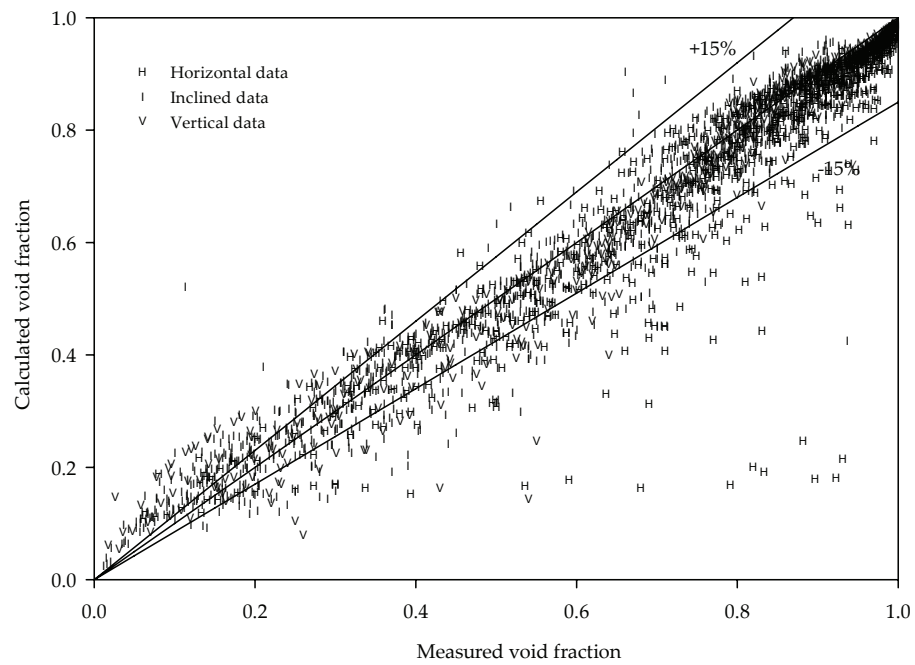


Figure 43. Comparison of Dix (1971) correlation with measured combined total experimental data

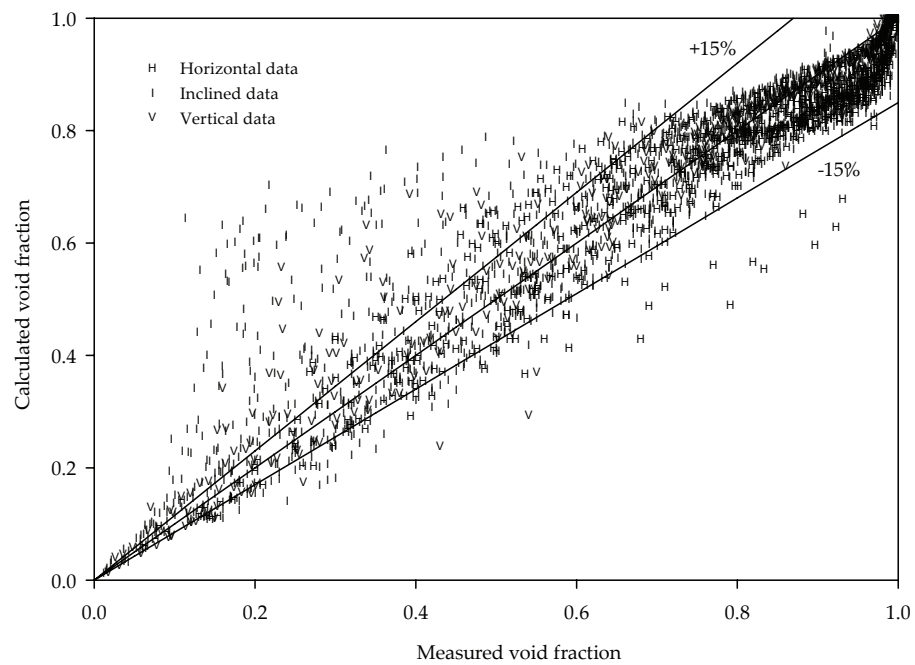


Figure 44. Comparison of Hughmark (1962) correlation with measured combined total experimental data

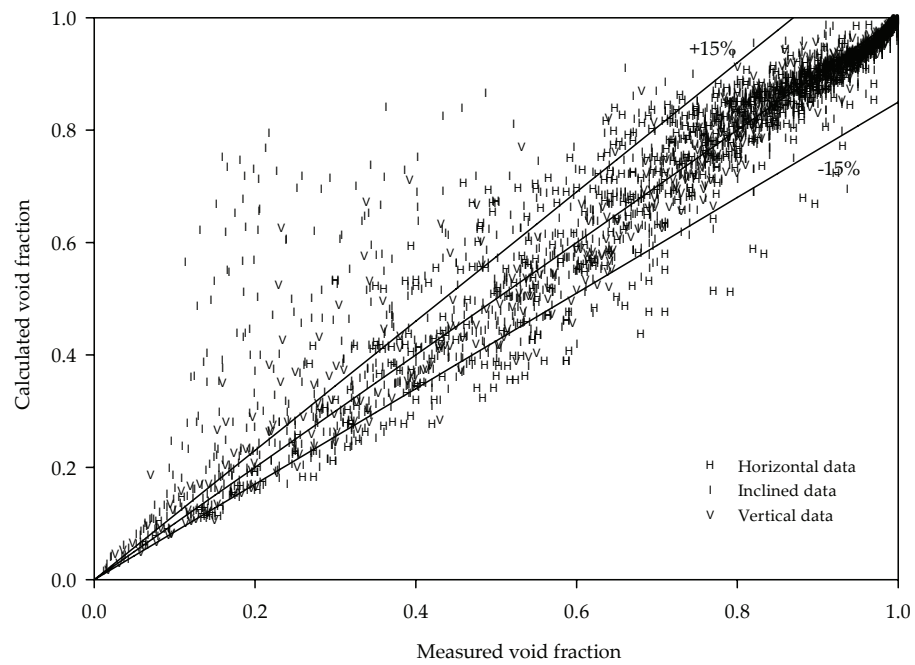


Figure 45. Comparison of Premoli et al. (1970) correlation with measured combined total experimental data

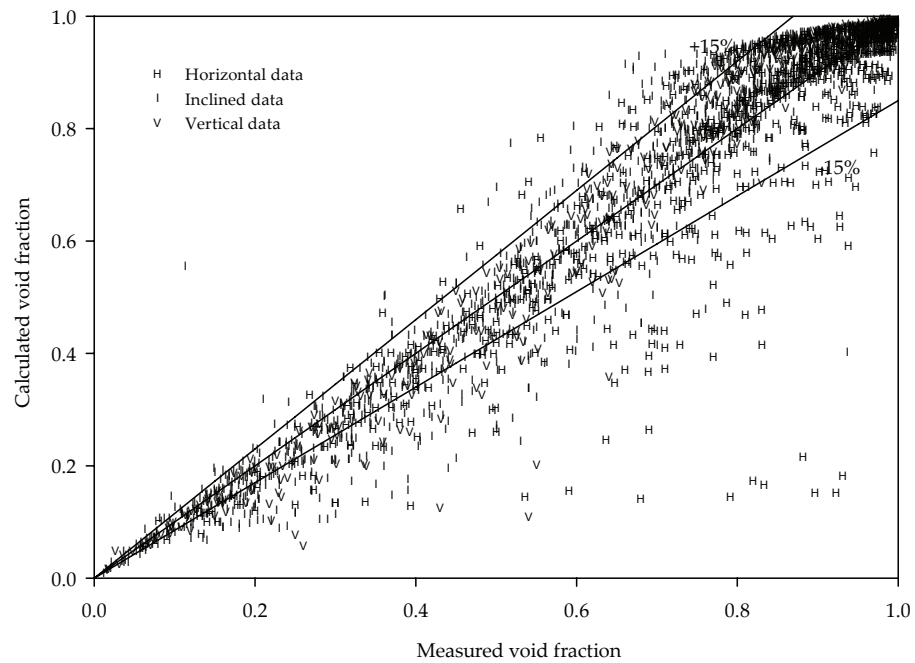


Figure 46. Comparison of Filimonov et al. (1957) correlation with measured combined total experimental data

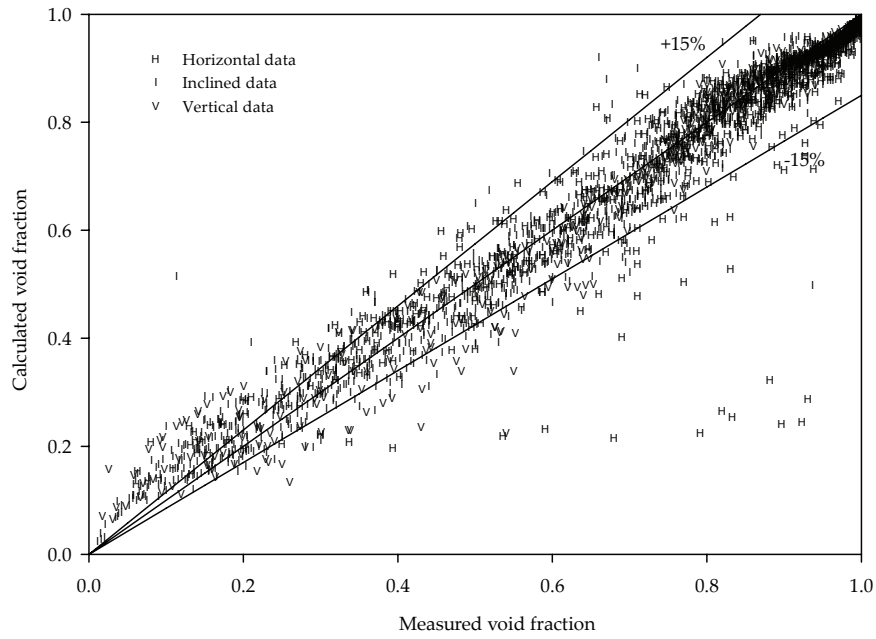


Figure 47. Comparison of present study correlation with measured combined total experimental data

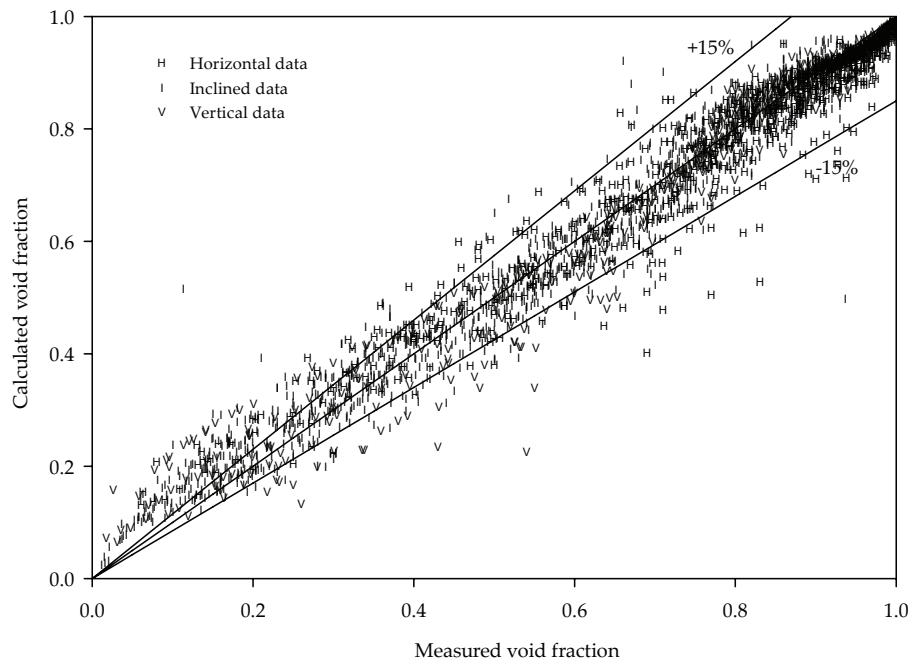


Figure 48. Comparison of present study correlation with measured “refined” combined total experimental data

## **CHAPTER 5**

### **CONCLUSIONS AND RECOMMENDATIONS**

#### **5.1 Conclusions**

A very extensive comparison of most of the void fraction correlations available in the open literature was made with data collected from different sources. The total number of data points considered was 2845 of which 900 were horizontal, 1542 inclined and 403 for vertical pipe setups. The best performing correlations within each of the three pipe orientations considered were given with detailed tables and plots showing their strengths and weakness with respect to inclination angle, fluid type and dataset.

A combined comparison for all the pipe inclinations from horizontal all the way up to vertical flow was done. The best performing correlations considering total number of data points predicted as well as their relative consistency in performance was highlighted. Appropriate recommendations were also given.

It was confirmed that the drift flux analysis method is a powerful tool in developing void fraction correlations as well as analyzing experimental data. Out of the six best performing correlations for the whole void fraction database, all except one were developed based on the drift flux model.

Detail study of the top performing correlations with each of the data points for which their performance deteriorated was carried out. An improvement to the correlation of Dix (1971) was made by systematically introducing appropriate physical parameters. It was shown that a total of 121 data points were additionally captured within the restrictive 5% index than that was predicted by the original correlation.

This improvement made the correlation to be the top correlation in predicting the whole database within each of the three error indices. The strong part of the modification done for the original correlation is that it is not a regression fit to the data with some very strange looking “lucky” constants but rather plausible physical arguments and parameters were forwarded and included. This would make it much more appealing to other correlations with fitted constants that would have to be changed if another data set were to be included in the analysis.

## **5.2 Recommendations**

The comparative analysis done on the collected void fractions is extensive both in the number of void fraction correlations considered and in the effort to include datasets from different sources. However, additional data is still required to make the conclusions more sound and valid. Some of the directions to which additional data collection should be geared towards and their justifications are:

1. The data base is more biased towards air water flow. Hence data with working fluids other than air-water would not only widen our data base but also would give definitive insight to the dependency of void fraction on physical properties like viscosity and surface tension.
2. The accuracy of some of the experimental data may be questionable especially for error percentage of  $\pm 5\%$ . Systematically controlled and accurate data within this range or even the confirmation of the fact that this is physically practicable with the methods currently used would be useful in “relieving” the burden from the correlations to predict the data within this range.
3. Experimental data from setups which operate close to atmospheric pressure and also with diameters less than 1 in. would be useful in validating some of



the dataset used in this analysis. It would also be used to validate and/or correct the correction factors introduced in the modified correlation.

4. Additional inclined experimental data with frequent angle intervals (5 degrees) would prove useful in filling some of the gaps in the database considered here. This would especially have a greater importance if the working fluids are different from air - water as was suggested in 1 above.
5. A similar comparative analysis of the correlations for downward flow is another area which may be further explored.
6. Taking into consideration the fact that there are some “gaps” in the inclined air-water experimental data set (refer to 2, 3 and 4 above), most of the best performing correlations didn’t predict one of the vertical datasets with a small diameter and the fact that there is no sufficient data for downward flow, an experimental test setup with a diameter of 0.375in which can cover inclination angles of  $-90^0$  all the way up to  $+90^0$  is built. This test setup is a multi purpose facility where heat transfer, pressure drop and void fraction measurements for single and two phase flows could be undertaken aided with flow visualization.

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## **APPENDIX A**

### **SUMMARY OF EXPERIMENTAL DATA BASE**

In this section a concise summary of the characteristics of each of the data set used in this work is given. This would include source of data set, pipe diameter, inclination angle, total data points in the set, the types of fluids used for the study and range of void fraction covered. Moreover, minimum and maximum values for parameters like pressure and temperature which are usually average values for both phases and mass or volumetric flow rate of each phase is also given. It is worth mentioning here that the calculation procedure in the comparison takes in the superficial velocities of the two phases as an input. Therefore, all mass or volumetric flow quantities would be converted to superficial velocities at the beginning of the calculations when evaluating the void fraction correlations. However, for reporting the data set here, no change is made in form to the original data set and everything would be reported as presented by the original researcher(s).

The complete and detailed data set is available from:

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**Data Source: Eaton (1966)**

**Inclination angle:** Horizontal

**Diameter of pipe:** 2.067 & 4.026 in (0.0525 & 0.10226m respectively)

**Total data points:** 237

**Fluids used:** natural gas – water

**Pressure range:** 305.30 – 868.70 *Psia*

**Temperature range:** 57.0 – 112.0  $^{\circ}F$

**Range of liquid volume flow rate:** 46.00– 5620.00 *bbl/d*

**Range of gas volume flow rate:** 36609.00 – 9126789.00 *scf/d*

**Range of void fraction covered:** 0.27 – 0.99

As mentioned in Chapter 3, Eaton (1966) reported the volume flow rate of natural gas in standard units (scf/d). Therefore, to convert this into the common volume flow rate units in  $m^3/sec$  or  $ft^3/s$ , the gas formation volume factor and the compressibility coefficient have to be calculated in addition to the basic unit conversions. The ranges of the calculated parameters and the converted volume flow rates are given below.

**Calculated input parameter ranges**

**Range of calculated compressibility factor:** 0.88– 0.96

**Range of calculated gas formation factor:** 0.56 – 1.78

**Range of converted liquid volume flow rate:** 0.0001– 0.01  $m^3/s$

**Range of converted gas volume flow rate:** 0.0005– 0.09  $m^3/s$

**Data Source: Beggs (1972)**

**Inclination angle:** Horizontal

**Diameter of pipe:** 1 & 1.5 in (0.0254 & 0.0381m respectively)

**Total data points:** 56

**Fluids used:** air – water

**Pressure range:** 51.92 – 98.6 *Psia*

**Temperature range:** 38 - 85  $^{\circ}F$

**Range of liquid phase flow rate:** 4.63 - 535.6 *lbm / s.ft<sup>2</sup>*

**Range of gas phase flow rate:** 0.48 – 25.41 *lbm / s.ft<sup>2</sup>*

**Range of void fraction covered:** 0.14 – 0.98

**Data Source: Beggs (1972)**

**Inclination angle:** 5 Degrees

**Diameter of pipe:** 1 & 1.5 in (0.0254 & 0.0381m respectively)

**Total data points:** 31

**Fluids used:** air – water

**Pressure range:** 65.70 – 98.97 *Psia*

**Temperature range:** 49 - 89  $^{\circ}F$

**Range of liquid phase flow rate:** 4.63 – 624.73 *lbm / s.ft<sup>2</sup>*

**Range of gas phase flow rate:** 0.49 – 25.53 *lbm / s.ft<sup>2</sup>*

**Range of void fraction covered:** 0.12 – 0.98

**Data Source: Beggs (1972)**

**Inclination angle:** 10 Degrees

**Diameter of pipe:** 1 & 1.5 in (0.0254 & 0.0381m respectively)

**Total data points:** 29

**Fluids used:** air – water

**Pressure range:** 65.45 – 98.51 *Psia*

**Temperature range:** 51 - 90  $^{\circ}F$

**Range of liquid phase flow rate:** 4.63 – 1065.09 *lbm / s.ft<sup>2</sup>*

**Range of gas phase flow rate:** 0.49 – 25.75 *lbm / s.ft<sup>2</sup>*

**Range of void fraction covered:** 0.14 – 0.98

**Data Source: Beggs (1972)**

**Inclination angle:** 15 Degrees

**Diameter of pipe:** 1 & 1.5 in (0.0254 & 0.0381m respectively)

**Total data points:** 31

**Fluids used:** air – water

**Pressure range:** 75.70 – 98.47 *Psia*

**Temperature range:** 53 - 90 <sup>0</sup>*F*

**Range of liquid phase flow rate:** 4.63 – 636.96 *lbm / s.ft<sup>2</sup>*

**Range of gas phase flow rate:** 0.53 – 29.75 *lbm / s.ft<sup>2</sup>*

**Range of void fraction covered:** 0.15 – 0.98

**Data Source: Beggs (1972)**

**Inclination angle:** 20 Degrees

**Diameter of pipe:** 1 & 1.5 in (0.0254 & 0.0381m respectively)

**Total data points:** 30

**Fluids used:** air – water

**Pressure range:** 64.49 – 98.75 *Psia*

**Temperature range:** 53 - 90 <sup>0</sup>*F*

**Range of liquid phase flow rate:** 4.63 – 1134.40 *lbm / s.ft<sup>2</sup>*

**Range of gas phase flow rate:** 0.49 – 24.97 *lbm / s.ft<sup>2</sup>*

**Range of void fraction covered:** 0.17 – 0.98

**Data Source: Beggs (1972)**

**Inclination angle:** 35 Degrees

**Diameter of pipe:** 1 & 1.5 in (0.0254 & 0.0381m respectively)

**Total data points:** 30

**Fluids used:** air – water

**Pressure range:** 81.67 – 97.32 *Psia*

**Temperature range:** 39 - 89 <sup>0</sup>*F*

**Range of liquid phase flow rate:** 4.63 – 517.21 *lbm / s.ft<sup>2</sup>*

**Range of gas phase flow rate:** 0.49 – 24.81 *lbm / s.ft<sup>2</sup>*

**Range of void fraction covered:** 0.19 – 0.98

**Data Source: Beggs (1972)**

**Inclination angle:** 55 Degrees

**Diameter of pipe:** 1 & 1.5 in (0.0254 & 0.0381m respectively)

**Total data points:** 29

**Fluids used:** air – water

**Pressure range:** 74.81 – 96.96 *Psia*

**Temperature range:** 46 - 92  $^{\circ}F$

**Range of liquid phase flow rate:** 4.63 – 632.12 *lbm / s.ft<sup>2</sup>*

**Range of gas phase flow rate:** 0.53 – 25.37 *lbm / s.ft<sup>2</sup>*

**Range of void fraction covered:** 0.20 – 0.98

**Data Source: Beggs (1972)**

**Inclination angle:** 75 Degrees

**Diameter of pipe:** 1 & 1.5 in (0.0254 & 0.0381m respectively)

**Total data points:** 29

**Fluids used:** air – water

**Pressure range:** 76.76 – 96.84 *Psia*

**Temperature range:** 51 - 95  $^{\circ}F$

**Range of liquid phase flow rate:** 4.62 – 624.43 *lbm / s.ft<sup>2</sup>*

**Range of gas phase flow rate:** 0.53 – 25.21 *lbm / s.ft<sup>2</sup>*

**Range of void fraction covered:** 0.15 – 0.98

**Data Source: Beggs (1972)**

**Inclination angle:** 90 Degrees

**Diameter of pipe:** 1 & 1.5 in (0.0254 & 0.0381m respectively)

**Total data points:** 26

**Fluids used:** air – water

**Pressure range:** 76.40 – 98.17 *Psia*

**Temperature range:** 67 - 96  $^{\circ}F$

**Range of liquid phase flow rate:** 4.62 – 516.78 *lbm / s.ft<sup>2</sup>*

**Range of gas phase flow rate:** 0.31 – 24.43 *lbm / s.ft<sup>2</sup>*

**Range of void fraction covered:** 0.15 – 0.98

**Data Source: Spedding & Nguyen (1976)**

**Inclination angle:** Horizontal

**Diameter of pipe:** 4.55 cm (0.0455m)

**Total data points:** 270

**Fluids used:** air – water

**Pressure range:** 747.84 – 860.52 *mm Hg* (absolute pressure)

**Temperature range:** 15.9 – 27.6 °C

**Range of liquid mass flow rate:** 6.01– 6093.40 *Kg/h*

**Range of gas mass flow rate:** 0.64 – 474.03 *Kg/h*

**Range of void fraction covered:** 0.06 – 1.00

**Data Source: Spedding & Nguyen (1976)**

**Inclination angle:** 2.75 degrees

**Diameter of pipe:** 4.55 cm (0.0455m)

**Total data points:** 226

**Fluids used:** air – water

**Pressure range:** 744.78 – 834.50 *mm Hg* (absolute pressure)

**Temperature range:** 18.20 – 23.70 °C

**Range of liquid mass flow rate:** 5.95– 8097.94 *Kg/h*

**Range of gas mass flow rate:** 0.06 – 523.71 *Kg/h*

**Range of void fraction covered:** 0.04 – 1.00

**Data Source: Spedding & Nguyen (1976)**

**Inclination angle:** 20.75 degrees

**Diameter of pipe:** 4.55 cm (0.0455m)

**Total data points:** 188

**Fluids used:** air – water

**Pressure range:** 711.94 – 876.58 *mm Hg* (absolute pressure)

**Temperature range:** 16.70 – 22.72 °C

**Range of liquid mass flow rate:** 5.85– 6089.19 *Kg/h*

**Range of gas mass flow rate:** 0.35 – 515.78 *Kg/h*

**Range of void fraction covered:** 0.03 – 1.00



**Data Source: Spedding & Nguyen (1976)**

**Inclination angle:** 45 degrees

**Diameter of pipe:** 4.55 cm (0.0455m)

**Total data points:** 196

**Fluids used:** air – water

**Pressure range:** 763.00 – 900.98 *mm Hg* (absolute pressure)

**Temperature range:** 16.13 – 24.33  $^{\circ}\text{C}$

**Range of liquid mass flow rate:** 5.75– 6093.91 *Kg/h*

**Range of gas mass flow rate:** 0.69 – 482.99 *Kg/h*

**Range of void fraction covered:** 0.04 – 1.00

**Data Source: Spedding & Nguyen (1976)**

**Inclination angle:** 70 degrees

**Diameter of pipe:** 4.55 cm (0.0455m)

**Total data points:** 279

**Fluids used:** air – water

**Pressure range:** 741.54 – 939.94 *mm Hg* (absolute pressure)

**Temperature range:** 17.60 – 23.60  $^{\circ}\text{C}$

**Range of liquid mass flow rate:** 5.88– 6098.97 *Kg/h*

**Range of gas mass flow rate:** 0.14 – 440.04 *Kg/h*

**Range of void fraction covered:** 0.01 – 0.99

**Data Source: Spedding & Nguyen (1976)**

**Inclination angle:** 90 degrees

**Diameter of pipe:** 4.55 cm (0.0455m)

**Total data points:** 224

**Fluids used:** air – water

**Pressure range:** 799.21 – 937.72 *mm Hg* (absolute pressure)

**Temperature range:** 16.35 – 23.73  $^{\circ}\text{C}$

**Range of liquid mass flow rate:** 32.41– 6095.97 *Kg/h*

**Range of gas mass flow rate:** 0.59 – 436.30 *Kg/h*

**Range of void fraction covered:** 0.04 – 0.99

**Data Source: Mukherjee (1979)**

**Inclination angle:** Horizontal

**Diameter of pipe:** 1.5 in (0.0381m)

**Total data points:** 62

**Fluids used:** air – kerosene

**Pressure range:** 28.20 – 91.90 *Psia*

**Temperature range:** 62 - 132  $^{\circ}F$

**Range of liquid superficial velocity:** 0.05 – 13.07 *ft/s*

**Range of gas superficial velocity:** 0.75 – 78.93 *ft/s*

**Range of void fraction covered:** 0.08 – 1.00

**Data Source: Mukherjee (1979)**

**Inclination angle:** 5 degrees

**Diameter of pipe:** 1.5 in (0.0381m)

**Total data points:** 57

**Fluids used:** air – kerosene

**Pressure range:** 26.70 – 68.70 *Psia*

**Temperature range:** 56 - 118  $^{\circ}F$

**Range of liquid superficial velocity:** 0.09 – 13.07 *ft/s*

**Range of gas superficial velocity:** 0.31 – 77.24 *ft/s*

**Range of void fraction covered:** 0.01 – 1.00

**Data Source: Mukherjee (1979)**

**Inclination angle:** 20 degrees

**Diameter of pipe:** 1.5 in (0.0381m)

**Total data points:** 74

**Fluids used:** air – kerosene

**Pressure range:** 26.90 – 72.90 *Psia*

**Temperature range:** 55 – 165.5  $^{\circ}F$

**Range of liquid superficial velocity:** 0.09 – 14.31 *ft/s*

**Range of gas superficial velocity:** 0.31 – 101.03 *ft/s*

**Range of void fraction covered:** 0.06 – 0.99

**Data Source: Mukherjee (1979)**

**Inclination angle:** 30 degrees

**Diameter of pipe:** 1.5 in (0.0381m)

**Total data points:** 74

**Fluids used:** air – kerosene

**Pressure range:** 27.70 – 88.70 *Psia*

**Temperature range:** 57 – 117.0  $^{\circ}F$

**Range of liquid superficial velocity:** 0.09 – 12.98 *ft/s*

**Range of gas superficial velocity:** 0.29 – 76.78 *ft/s*

**Range of void fraction covered:** 0.09 – 0.99

**Data Source: Mukherjee (1979)**

**Inclination angle:** 50 degrees

**Diameter of pipe:** 1.5 in (0.0381m)

**Total data points:** 67

**Fluids used:** air – kerosene

**Pressure range:** 23.90 – 93.70 *Psia*

**Temperature range:** 40 – 127.0  $^{\circ}F$

**Range of liquid superficial velocity:** 0.05 – 11.33 *ft/s*

**Range of gas superficial velocity:** 0.15 – 79.34 *ft/s*

**Range of void fraction covered:** 0.03 – 1.00

**Data Source: Mukherjee (1979)**

**Inclination angle:** 70 degrees

**Diameter of pipe:** 1.5 in (0.0381m)

**Total data points:** 80

**Fluids used:** air – kerosene

**Pressure range:** 40.20 – 83.70 *Psia*

**Temperature range:** 44.5 – 122.0  $^{\circ}F$

**Range of liquid superficial velocity:** 0.06 – 11.09 *ft/s*

**Range of gas superficial velocity:** 0.14 – 89.98 *ft/s*

**Range of void fraction covered:** 0.14 – 0.99

**Data Source: Mukherjee (1979)**

**Inclination angle:** 80 degrees

**Diameter of pipe:** 1.5 in (0.0381m)

**Total data points:** 92

**Fluids used:** air – kerosene

**Pressure range:** 40.20 – 87.20 *Psia*

**Temperature range:** 60.0 – 113.0  $^{\circ}F$

**Range of liquid superficial velocity:** 0.06 – 11.27 *ft/s*

**Range of gas superficial velocity:** 0.19 – 66.43 *ft/s*

**Range of void fraction covered:** 0.17 – 0.99

**Data Source: Mukherjee (1979)**

**Inclination angle:** 90 degrees

**Diameter of pipe:** 1.5 in (0.0381m)

**Total data points:** 52

**Fluids used:** air – kerosene

**Pressure range:** 38.70 – 88.40 *Psia*

**Temperature range:** 54.0 – 118.0  $^{\circ}F$

**Range of liquid superficial velocity:** 0.09 – 10.97 *ft/s*

**Range of gas superficial velocity:** 0.12 – 80.08 *ft/s*

**Range of void fraction covered:** 0.02 – 0.98

**Data Source: Minami & Brill (1987)**

**Inclination angle:** Horizontal

**Diameter of pipe:** 3.068 in (0.07793m)

**Total data points:** 54

**Fluids used:** air – water

**Pressure range:** 46.40 – 85.40 *Psia*

**Temperature range:** 76 - 117  $^{\circ}F$

**Range of liquid superficial velocity:** 0.02 – 2.96 *ft/s*

**Range of gas superficial velocity:** 1.56 – 49.13 *ft/s*

**Range of void fraction covered:** 0.55 – 0.99

**Data Source: Minami & Brill (1987)**

**Inclination angle:** Horizontal

**Diameter of pipe:** 3.068 in (0.07793m)

**Total data points:** 57

**Fluids used:** air – kerosene

**Pressure range:** 43.70 – 96.70 *Psia*

**Temperature range:** 82 - 118  $^{\circ}F$

**Range of liquid superficial velocity:** 0.02 – 3.12 *ft/s*

**Range of gas superficial velocity:** 1.78 – 54.43 *ft/s*

**Range of void fraction covered:** 0.56 – 0.99

**Data Source: Franca & Lahey (1992)**

**Inclination angle:** Horizontal

**Diameter of pipe:** 0.019m

**Total data points:** 81

**Fluids used:** air – water

**Pressure range:** 0.0 – 1.47 *m H<sub>2</sub>O* (Gauge Pressure)

**Temperature range:** 22 - 22 <sup>0</sup>*C*

**Range of liquid superficial velocity:** 0.01 – 1.49 *m / s*

**Range of gas superficial velocity:** 0.13 – 23.76 *m / s*

**Range of void fraction covered:** 0.06 – 0.94

**Data Source: Abdulmajeed (1996)**

**Inclination angle:** Horizontal

**Diameter of pipe:** 2 in (0.0508m)

**Total data points:** 83

**Fluids used:** air – kerosene

**Pressure range:** 197.20 – 919.10 *KPa*

**Temperature range:** 27.8 – 48.9  $^{\circ}C$

**Range of liquid superficial velocity:** 0.002 – 1.83 *m/s*

**Range of gas superficial velocity:** 0.20 – 48.91 *m/s*

**Range of void fraction covered:** 0.39 – 0.99

**Data Source: Sujumnong (1997)**

**Inclination angle:** 90 degrees

**Diameter of pipe:** 0.5 in (0.0127m)

**Total data points:** 101

**Fluids used:** air – water

**Pressure range:** 101351.35 – 342664.09 *Pa*

**Temperature range:** 18.97 – 32.13  $^{\circ}\text{C}$

**Range of liquid superficial velocity:** 0.05– 8.46 *m / s*

**Range of gas superficial velocity:** 0.04 – 118.36 *m / s*

**Range of void fraction covered:** 0.02 – 0.99



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Pages in Study: 160

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Scope and Method of Study: An extensive literature search was made for the available void fraction correlations and experimental void fraction data. After systematically refining the data, the performance of the correlations in correctly predicting the diverse data sets was evaluated. Comparisons between the correlations were made and appropriate recommendations drawn. An improved version of one of the top performing correlations was also suggested by introducing physical parameters backed by plausible arguments that capture the physics of the problem.

Findings and Conclusions: The analysis showed that most of the correlations developed are very restricted in terms of handling a wide variety of data sets. Based on the recommendations made, an improved void fraction correlation which could acceptably handle all data sets regardless of flow patterns for horizontal and upward inclination angles was suggested. It was shown that this correlation has the best predictive capability than all the correlations considered in this study.

Advisor's Approval: Dr. Afshin J. Ghajar